Out-of equilibrium bids in first-price auctions: Wrong expectations or wrong bids*

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Abstract

Deviations from risk-neutral equilibrium bids in auctions can be related to inconsistent expectations with correct best replies or correct expectations but deviant best replies (e.g. due to risk aversion, regret, quantal-response mistakes). To distinguish between these two explanations we use a novel experimental procedure and study expectations together with best replies in symmetric and asymmetric auctions. We extensively test the internal validity of this setup. We find that deviations from equilibrium bids do not seem to be due to wrong expectations but due to deviations from a risk-neutral best reply.

Keywords: Experiments, Auction, Expectations.

(JEL C92, D44)

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1 Introduction

Since early auction experiments by Coppinger et al. (1980) and Cox et al. (1982) it is well-known and repeatedly confirmed that bidders in auctions consistently deviate from risk neutral symmetric Bayesian Nash equilibrium (RNBNE). We broadly distinguish three different approaches to explain deviating bidding behaviour: (1) Some authors drop the equilibrium concept entirely and replace it with a dynamic concept like learning (see Ockenfels and Selten, 2005; Neugebauer and Selten, 2006). (2) A large part of the literature keeps the notion of an equilibrium but modifies the best-reply behaviour of players. This can be done either explicitly, e.g. through introducing risk aversion or inequity aversion or regret, or implicitly through allowing for random mistakes in the best-reply. (3) A smaller part of the literature studies equilibria where players do not always form correct expectations. In this paper we want to distinguish between the second and the third approach.

Let us briefly review the literature on the second approach: modifications of the best-reply behaviour. This literature assumes that players follow a Bayesian Nash equilibrium but replaces, e.g., risk neutrality with risk aversion. Indeed, risk aversion explains overbidding deviations from RNBNE to some degree in first-price auctions (see, e.g., Andreoni et al., 2007; Chen and Plott, 1998; Cox et al., 1988; Kirchkamp et al., 2006). However, risk aversion does not explain all deviations addressed by the literature on auctions. E.g., overbidding in second-price auctions (see Kagel et al., 1987; Harstad, 2000; Cooper and Fang, 2008) or overbidding in third price auctions (see Kagel and Levin, 1993) cannot be explained by risk aversion. Furthermore, individual attitudes toward risk are not consistent over different institutions (see Isaac and James, 2000). Accordingly, recent literature suggests alternative modifications of the utility function, introducing motives like regret (see Filiz-Ozbay and Ozbay, 2007; Engelbrecht-Wiggans and Katok, 2007a), or spite (Morgan et al., 2003). Furthermore, Goeree et al. (2002) introduce quantal-response-equilibria and explain overbidding in first-price private-value auctions as the result of small mistakes in forming best replies.

So far we have confined our attention to the specific way bidders choose their best replies. However, unexpected or unexplained bids can also be due to inconsistent expectations of the bidders (see Stahl and Wilson, 1995). Indeed Chen and Plott (1998, p. 73) suggest for first-price auctions that "the theory of beliefs and belief formation might be the most productive place to work." Following this advice, Eyster and Rabin (2005) and Crawford and Iriberri (2007) show that simplified (and inconsistent) expectations can explain overbidding in first-price common-value auctions. For first-price private-value auctions, Goeree et al. (2002, p. 263) demonstrate that misperceived probabilities of winning the auction would explain overbidding as well as risk aversion.

Obviously, we can fit experimental data either with suitable mistakes in expectations or with manipulated best replies. Experiments that we mentioned so far observe only bids and can, thus, not distinguish these two approaches. In this paper we introduce and test a new method which allows us to measure expectations about bidding strategies of other bidders and bids together. This method allows us to gain more insight into the bidding process and to better understand to which degree expectations and best replies contribute to deviations from risk neutral Bayesian Nash equilibria.

While expectations have been elicited in several experimental studies before, this is the first study to elicit expectations about the strategies of other players in the context of auctions. The literature on incentivised belief elicitation addresses various settings including public goods (e.g. Croson, 2000; Gächter and Renner, 2010; Offerman et al., 1996, 2001; Wilcox and Feltovich, 2000), normal-form games (e.g. Costa-Gomes and Weizsäcker, 2008; Ehrblatt et al., forthcoming; Fehr et al., 2008; Haruvy, 2002; Ivanov, forthcoming; Mason and Phillips, 2001; Nyarko and Schotter, 2002; Rey-Biel, 2009), various market settings (e.g. Haruvy et al., 2007; Hommes et al., 2005; Marimon and Sunder, 1993; Sonnemans et al., 2004), information pooling (e.g. McKelvey and Page, 1990), trust games (e.g. Dufwenberg and Gneezy, 2000), information cascades (e.g. Dominitz and Hung, 2004) and individual choice tasks (e.g. Kelley and Friedman, 2002; Schmalensee, 1976).

In the context of first-price private-value auctions, Armantier and Treich (2009) elicit expectations about the probabilities of winning the auction with some bid. This is complementary to our approach of eliciting expectations about bidding strategies. Interestingly, they find that, except for very small and very large bids, reported expectations about probabilities of winning the auction are lower than actual probabilities implied by actual bidding behaviour in their experiment.

Expectations[1] and bids can be measured either simultaneously or in isolation. In this paper we pursue both approaches. In the simultaneous treatment participants play together in pairs. Each player in a pair forms expectations about her opponent's strategy. We elicit these expectations. Based on these expectations, players submit bids. We can then compare bids with expectations and find out to what degree deviations from Bayesian Nash equilibrium bids are a result of erroneous bids or a failure to behave optimally given these bids.

In an alternative treatment we observe single players who play against a comput-

[1] Here and in the remainder of this paper we use the short-hand 'expectations' to refer to 'expectations about the bidding strategies of other bidders'.

erised opponent with an announced bidding function. These bidding functions are those of human bidders from previous experiments. With an announced strategy of the opponent it is not difficult for bidders to form expectations. We can observe bids isolated from expectation formation. A similar approach is used by Walker et al. (1987) who study an experiment where participants bid against a computerised opponent. In their experiment participants are not informed about the computerised bidding function, thus, a comparison of bids with best replies is not possible. Neugebauer and Selten (2006) and Dorsey and Razzolini (2003) study experiments with first-price private-value auctions where bidders play against computerised opponents with a known distribution of bids. Charness and Levin (2009) also use computerised opponents in a common value setting. Such a setup allows to compare participants' strategies with best replies.

Some experimental studies, (e.g. Costa-Gomes and Weizsäcker, 2008; Offerman et al., 1996), suggest that strategies are typically not best responses to held expectations while expectations seem to resemble actual strategies fairly well. In comparison to these experiments, we analyse a completely different type of game (an auction with incomplete information and infinitely many actions).

We briefly summarise the equilibrium model in section 2. The experimental treatments are discussed in section 3 and internal validity of our method is checked in section 4. We present results in section 5 and conclude in section 6.

2 Model

Our workhorse is a private value first-price sealed-bid auction with two bidders i and j. This auction type is simple and still allows us to describe expectations and best replies in a non-trivial way. It might be interesting to enrich this environment by introducing common values in a later study. Here, however, we prefer the simplicity of the private value setting.

2.1 Symmetric Auction

In our experiment we look at two situations: One where valuations are symmetrically distributed for both bidders, and one where values follow different distributions for both bidders. Let us start with the symmetric case. Bidders' valuations x_i and x_j are independently distributed according to a distribution function $F()$ which is the same for each bidder. The derivation of risk neutral symmetric Bayesian Nash equilibria is standard and reported to introduce notation. We rely on risk neutral equilibria as a benchmark and we use an experimental setup that eliminates a substantial part of the risk that bidders face in auctions. Bidder i with valuation x_i expects the opponent to follow a monotonically increasing bidding function $b^{\exp}(x_j)$ with inverse $b^{\exp(-1)}(\cdot)$. If bidder i bids $b(x_i)$ then this bidder gains $x_i - b(x_i)$ with probability $F(b^{\exp(-1)}(b(x_i)))$ and the expected profit is $u = (x_i - b(x_i)) \cdot F(b^{\exp(-1)}(b(x_i)))$. Bidders choose their individual bidding function b_i to maximise u given their expected opponents' bidding function $b^{\exp}$. It is straightforward to show (Vickrey, 1961) that if $F()$ is a uniform distribution over some interval $[0, \bar{x}]$ both bidders have a symmetric bidding function

$$b^*(x) = \frac{1}{2}x \tag{1}$$

in the symmetric equilibrium. We should note that, while there are auction situations where further asymmetric equilibria exist, the unique equilibrium in the introduced auction model is symmetric (Maskin and Riley, 2003).

2.2 Asymmetric auction

While a symmetric auction is perhaps simpler to understand for participants, the symmetric setting makes it also harder for us to observe the direction of the causality. Do bidders really first form expectations and then optimise against these expectations? One might fear that the idea of describing expectations is so abstract that bidders use a simple heuristic: First they determine a strategy and then, when asked to state expectations, they infer expectations about the behaviour of the other bidder from their own bids? One way to address this issue is to introduce auctions where values are distributed differently for both bidders and where, hence, also bids should differ.

We obtain the asymmetric auction case by contracting bidder i's support while stretching the one of bidder j such that lower bounds remain fixed. Again, bidders' valuations x_i and x_j are independently distributed. Specifically, the cumulative distribution functions are given by $F_i : [\underline{x}, \overline{x}_i] \rightarrow [0, 1]$ and $F_j : [\underline{x}, \overline{x}_j] \rightarrow [0, 1]$ such that $\overline{x}_j > \overline{x}_i$. For a theoretical analysis of asymmetric auctions with private values see Plum (1992) and Maskin and Riley (2000).

In accordance with our experimental setup we assume uniformly distributed valuations and refer to bidder i as the weak bidder and to bidder j as the strong bidder since F_i is first-order stochastically dominated by F_j. For our case of two risk neutral bidders with uniformly distributed valuations, Plum (1992) shows that there exists a unique Bayesian Nash equilibrium and provides explicit equilibrium bidding functions. Denoting the equilibrium bidding function of the weak bidder by $b_w^*(x)$ and that of the strong bidder by $b_s^*(x)$, with $c = (\overline{x}_i - \underline{x})^{-2} - (\overline{x}_j - \underline{x})^{-2}$, we obtain for our asymmetric

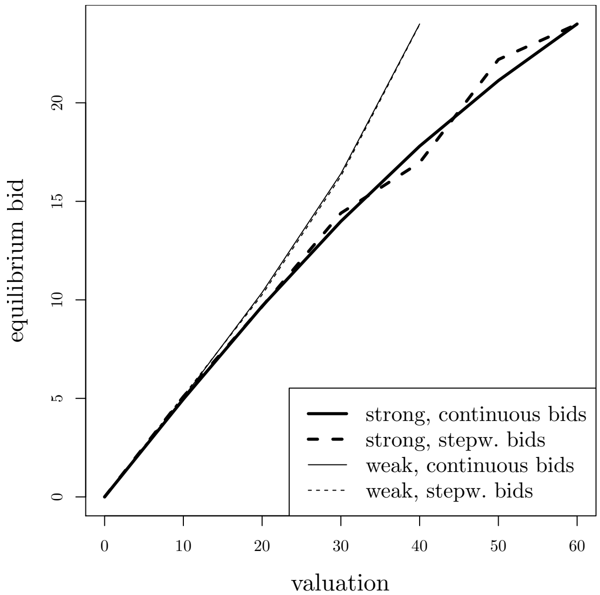


Figure 1: Equilibrium bids for bidders with asymmetric valuations

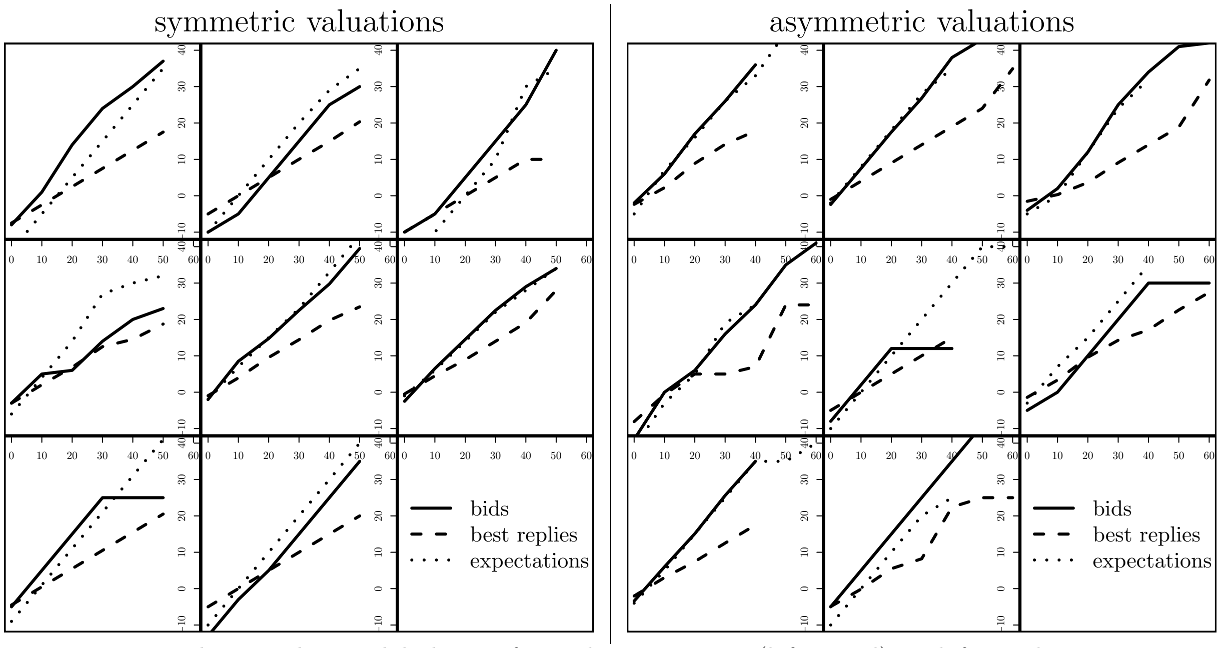
The solid line shows equilibrium bids for the case where bidders can choose any differentiable bidding function. The dashed lines show equilibrium bids for bidders who can only choose a stepwise linear bidding function as in our experiment. Valuations are normalized to the intervals of $[0, 40]$ and $[0, 60]$ respectively.

auction:

$$b_w^*(x) = \underline{x} + \frac{1 - \sqrt{1 - c\,(x - \underline{x})^2}}{c\,(x - \underline{x})} \qquad (x \in [\underline{x}, \overline{x}_i]) \tag{2}$$

$$b_s^*(x) = \underline{x} + \frac{\sqrt{1 + c\,(x - \underline{x})^2} - 1}{c\,(x - \underline{x})} \qquad (x \in [\underline{x}, \overline{x}_j]) \tag{3}$$

Figure 1 depicts the equilibrium bidding functions for the parametrization utilized in our experiment where $\underline{x} = 50$, $\overline{x}_i = 90$ and $\overline{x}_j = 110$. The solid lines in the figure are, however, not exactly the equilibrium bids in our experiment. To keep things simple we allow for only stepwise linear bids in the experiment. This is no serious problem in the symmetric case since there equilibrium bids are a linear function. It is, however, a problem in the asymmetric case. With the stepwise linear bids we use in the experiment the equilibrium bids are the dashed lines shown in the figure. Whenever we refer to equilibrium bids below we take this restriction into account.



Expectations, best replies and bids are for eight symmetric (left panel) and for eight asymmetric bidders (right panel) in round 7 of two sessions on 12 May 2005 and on 2 November 2009 respectively. A bidder's type in the asymmetric treatment, weak or strong, can be inferred from the domain of the best reply schedules that are indicated as dashed lines. For weak bidders the domain of valuations is $[0, 40]$ and for the strong bidders it is $[0, 60]$.

Figure 2: Examples for expectations, best replies, and bids

2.3 Expectations and best replies

The derivation of equilibrium bids distinguishes between two steps of reasoning independent of the symmetric or the asymmetric auction case. First, bidders form expectations, b^{exp} in the symmetric auction and b_w^{exp} or b_s^{exp} in the asymmetric auction, about the bidding function of their opponent; in the risk neutral Bayesian Nash equilibrium we have $b^{\text{exp}} = b^*$ in the symmetric auction and $b_w^{\text{exp}} = b_s^*$ and $b_s^{\text{exp}} = b_w^*$ in the asymmetric auction. Then bidders determine a best reply, $b^{\text{opt}|\text{exp}}$ in the symmetric auction and $b_w^{\text{opt}|\text{exp}}$ or $b_s^{\text{opt}|\text{exp}}$ in the asymmetric auction, given these expectations and play this best reply; in equilibrium also $b^{\text{opt}|\text{exp}} = b^*$ or $b_w^{\text{opt}|\text{exp}} = b_w^*$ and $b_s^{\text{opt}|\text{exp}} = b_s^*$. Figure 2 shows some examples of expected opponent's bidding functions b^{exp} observed in our experiment together with the hypothetical best reply $b^{\text{opt}|\text{exp}}$, and the bids b actually submitted in the experiment. The left graph shows a treatment with symmetric valuations, the graph on the right shows a treatment with asymmetric valuations. The examples illustrate a general property: In the experiment bids b clearly differ from best replies $b^{\text{opt}|\text{exp}}$.

In section 3 we describe an experiment that allows us to observe the two steps of this decision process, i.e. expectations (which imply best replies) together with actual bids. Once we observe these two steps together, we can better understand why bids deviate from risk neutral Bayesian Nash equilibrium bids. We will be able to distinguish between two types of bidders: bidders who form expectations which are systematically wrong but whose best replies against these expectations are correct (similar to the bidders proposed by Crawford and Iriberri (2007)), and bidders with correct expectations who submit bids which are not risk-neutral best replies.

With this exercise we do not aim to provide a complete and correct description of the thought process of real individuals. Instead we follow the structure of equilibrium derivation within the context of expected utility theory. Therefore, we can only find out where the standard equilibrium model of bidding behaviour provides a good approximation of human behaviour and where it does not. By decomposing this model into two steps we can, however, learn more than by only observing bids without expectations.

2.4 Level-k reasoning

By observing bids and expectations together we can also take another look at models of iterated strategic reasoning introduced by Stahl and Wilson (1995) and Nagel (1995) and applied to auctions by Crawford and Iriberri (2007). There are two interpretations of the iterated strategic reasoning model that differ in the response of a level-k type to the lower-level types $L(k-1)$,..., $L0$. While Stahl and Wilson (1995) and Camerer et al. (2004) assume a level-k type to best-reply to a distribution of all lower-level types, Nagel (1995), Costa-Gomes et al. (2001) and Crawford and Iriberri (2007) assume a level-k type to best-reply to the level-$(k-1)$ type only. In the following we briefly report how the level-k types behave in our symmetric and asymmetric auctions under either interpretation noting that not much can be gained from assuming responses to a non-degenerate distribution of lower levels in our setting.[2]

2.4.1 Level-k playing level-$(k-1)$ only

Following Crawford and Iriberri (2007) when moving from a complete information setting to an incomplete information setting, the simplest player in their model of level-k thinking, the $L0$ player, is the starting point of a player's strategic reasoning. If this player is 'random', the player chooses all bids between the smallest possible valuation and the highest possible valuation with equal probability. If this player is 'truthful', the player always bids the own valuation. With our distribution of valuations both such types imply the same uniform distribution of bids. It is straightforward to show that equation (1) describes the best reply if expected bids are uniformly distributed on

[2]For the derivations of the level-k best replies see appendix A

support $[0, \overline{b}]$ and if the best-reply according to equation (1) does not exceed $\overline{b}$.

For our symmetric auction, it immediately follows that the best reply of $L1$ against an $L0$ player is the bid given by equation (1). $L0$ players choose all bids from their possible range with equal probability. $L1$ players do the same for their expectations but choose a best reply against $L0$ (that coincides with the equilibrium bid). $L2$ and higher order players have expectations and bids given by equation (1). Thus, our experiment allows to distinguish between $L0$, $L1$, and $L2$ and higher order players in the symmetric setting.

In the asymmetric setting a weak bidder competes with a strong bidder leading to two variants of any Lk: weak Lk and strong Lk. Due to the asymmetric nature of the auction weak Lk best-replies to strong $L(k-1)$ and strong Lk best-replies to weak $L(k-1)$. Analogously to the reasoning in the symmetric auction, weak $L1$ and strong $L1$ expect uniformly distributed bids on the valuation range of strong $L0$ and weak $L0$ players, respectively, leading to best replies given by (1). The same reasoning applies to weak $L2$ players, but there is a twist for strong $L2$ players. There is a continuum of strong $L2$ types that would, if following (1), submit suboptimal bids exceeding the largest bid of the competitor $\overline{b}_{wL1}$. For these types a boundary solution emerges where it is optimal to bid $\overline{b}_{wL1}$. As a result, strictly positive probability mass concentrates at $\overline{b}_{wL1}$ while probability density is distributed uniformly on $[0, \overline{b}_{wL1})$. With continuous bids, there does not exist a best reply to strong $L2$ bidding due to the discontinuity of the cumulative distribution function of expected bids so that bids are not defined for weak $L3$. Bids and expectations of strong $L3$ players are well-defined and coincide with those of strong $L2$ players but do not exist beyond this level. Therefore, both variants of $L0$, $L1$, and $L2$ bidders behave in the same way as if playing the symmetric auction except of strong $L2$ bidders with a value of $x \geq \overline{x}_{\mathrm{w}}$ bidding $\overline{b}_{wL1}$.[3]

2.4.2 Level-k playing a lower level distribution

In the symmetric auction, the best replies of any level Lk_{mix} [4] are indistinguishable from the best replies of Lk. Both variants of the level-k-model differ in the expectations held by types of levels $L2$ and higher only.[5] While levels $L2+$ believe to play against risk-neutral equilibrium bids for sure as outlined above, levels $L2+_{\mathrm{mix}}$ believe to play against $L0$-bids with strictly positive probability and to play against risk-neutral equilibrium

[3] The emergence of a discontinuous cumulative distribution function of bids is independent of the parameterisation of the asymmetric auction and occurs with uniform distributions either at level strong $L1$ or latest, as in our case, at the level of strong $L2$.

[4] We use the subscript 'mix' to identify the levels when playing against a distribution of lower levels.

[5] For simplicity we denote the statement 'levels Lk and higher' by $Lk+$.

bids submitted by any level $L1+_{\text{mix}}$ with the residual probability. Further, levels $L1+_{\text{mix}}$ differ from one another in the probability of expecting to face $L0$-bids only due to indistinguishable best reply functions of any type higher than $L0_{\text{mix}}$.

In the asymmetric auction, the assumption of playing against a distribution of lower levels does not affect best replies much either. The only difference in bidding applies to strong $L2_{\text{mix}}$ (and likewise to strong $L3_{\text{mix}}$) if the probability of facing $L0$-bids, λ, is rather large, $\lambda \in (2/3, 1)$. In this case the flat part of the best reply function of strong $L2$ (hencforth $sL2$) "shortens" and is joined by a strictly increasing part. Due to the flat part, probability mass concentrates on a single bid so that the best replies for higher level are not defined here, too. Analogously to the case of the symmetric auction, expectations of levels weak/strong $L1+_{\text{mix}}$ only differ from one another in the probability of expecting to face $L0$-bids or risk-neutral equilibrium bids.

3 Experimental setup

We use the strategy method to observe bidding functions in a way similar to Selten and Buchta (1999), Güth et al. (2003), Pezanis-Christou and Sadrieh (2003), Kirchkamp and Reiß (2004), and Kirchkamp et al. (2009). Other experiments that use this method (see Kirchkamp et al., 2009; Kirchkamp and Reiß, 2004) show that bidding behaviour that is observed with the strategy method is very similar to the behaviour observed with alternative methods. The strategy method allows us to observe bidding functions in much more detail. More importantly, it lends itself also to observe expectations.

We compare five treatments. The first four of them base on symmetric valuations, the fifth one uses asymmetric valuations:

no expectations In this treatment we only elicit bids. This is our baseline treatment. The only payoff in the treatment is the profit in the auctions.

expectations In this treatment we elicit bids and expectations. The payoff in this treatment is the profit in the auctions plus a reward for precision of expectations.

expectations with info Here we elicit bids and expectations and give feedback about the precise bidding function of the opponents. As in the previous treatment the payoff in this treatment is the profit in the auctions plus a reward for precision of expectations.

computerised opponents In this treatment bidders do not compete against humans but against a computer with an announced bidding function. In each round the

	independent observations	participants
asymmetric	8	70
computer opp.	17	17
expectations	8	74
expectations w. info	11	102
no expectations	12	116
all	56	379

Table 1: Overview of treatments

computer uses a new bidding function that is taken from a randomly selected player who participated in the 'expectations with info' treatment that we conducted earlier.

asymmetric In this treatment the distribution of valuations differs across bidders. Everything else is analogous to the 'expectations with info' treatment.

All experiments were conducted between 12/2003 and 11/2009 in the experimental laboratories at the SFB 504 in Mannheim, at MaXLab in Magdeburg, and at the University of Jena. A total of 379 subjects participated in these experiments. The average profit of a participant was 10.46€ with a standard deviation of 4.64€.

Table 1 gives an overview. A detailed list of the sessions is provided in appendix B, the experimental procedure is described in appendix E. The software we used was z-Tree Versions 3α and 3.3.6 (the final version is documented in Fischbacher, 2007). In each treatment subjects first received written instructions, then they answered a quiz on the computer screen to make sure that they understood the instructions. Thereafter, they played twelve rounds of the actual experiment. In each of these rounds participants were matched randomly in groups of two. Each group participated in five simultaneous auctions. All treatments concluded with a questionnaire and the payment of subjects in cash.

Input of bidding functions: This stage was common to all treatments. Subjects would submit their own bidding function and, in the treatment with expectations, the expected bidding function of their opponent. The smallest valuation in the experiment was always $\underline{x} = 50$. The largest valuation was $\bar{x} = 100$ for the symmetric treatments. In the asymmetric treatments we had $\bar{x} = 90$ for the weak and $\bar{x} = 110$ for the strong bidder. In each round participants enter bids for six valuations with symmetric bidders and for four or seven valuations with asymmetric bidders. All valuations are equally spaced between $\underline{x}$ and $\bar{x}$ in steps of 10. Bids for all other valuations are interpolated linearly.

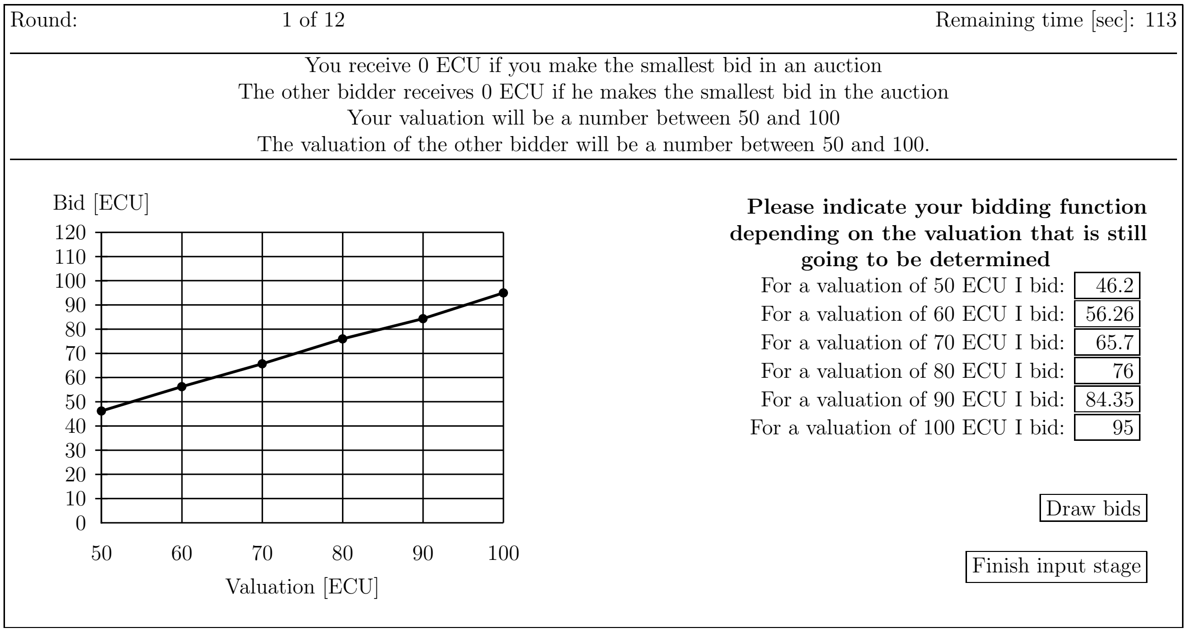


Figure 3: Stage 1: A typical input screen in the 'no expectations' treatment (translated into English)

A typical input screen for the 'no expectation' treatment is shown in figure 3. A typical input screen for the treatments with expectations is shown in figure 4. The input screen for the 'computerised opponents' treatment is similar to that of the 'expectations' treatment depicted in figure 4 where the selected bidding function used by the computerised opponent is announced numerically and graphically on the right side of the screen, replacing the expectation elicitation area.

In this paper we always discuss normalised valuations where the smallest valuation is 0 and the largest valuation is $\bar{x} - \underline{x}$.

Auction feedback: When all participants have determined their bidding functions they move on to the auction feedback stage. In this stage they play five independent auctions, i.e. the computer draws five pairs of random and independent valuations for each pair of participants. In each of these five independent auctions the winner is determined and the profit of each player is calculated. The sum of the profit of these five auctions is the total auction profit gained in this round. We play five auctions for two reasons: First, multiple auctions and, hence, multiple valuations, may help participants to think carefully about all parts of their bidding function. Second, and more importantly, playing multiple auctions reduces a substantial part of risk. Kirchkamp, Reiß, and Sadrieh (2006) systematically explore the approach of playing multiple auctions with a given bidding function

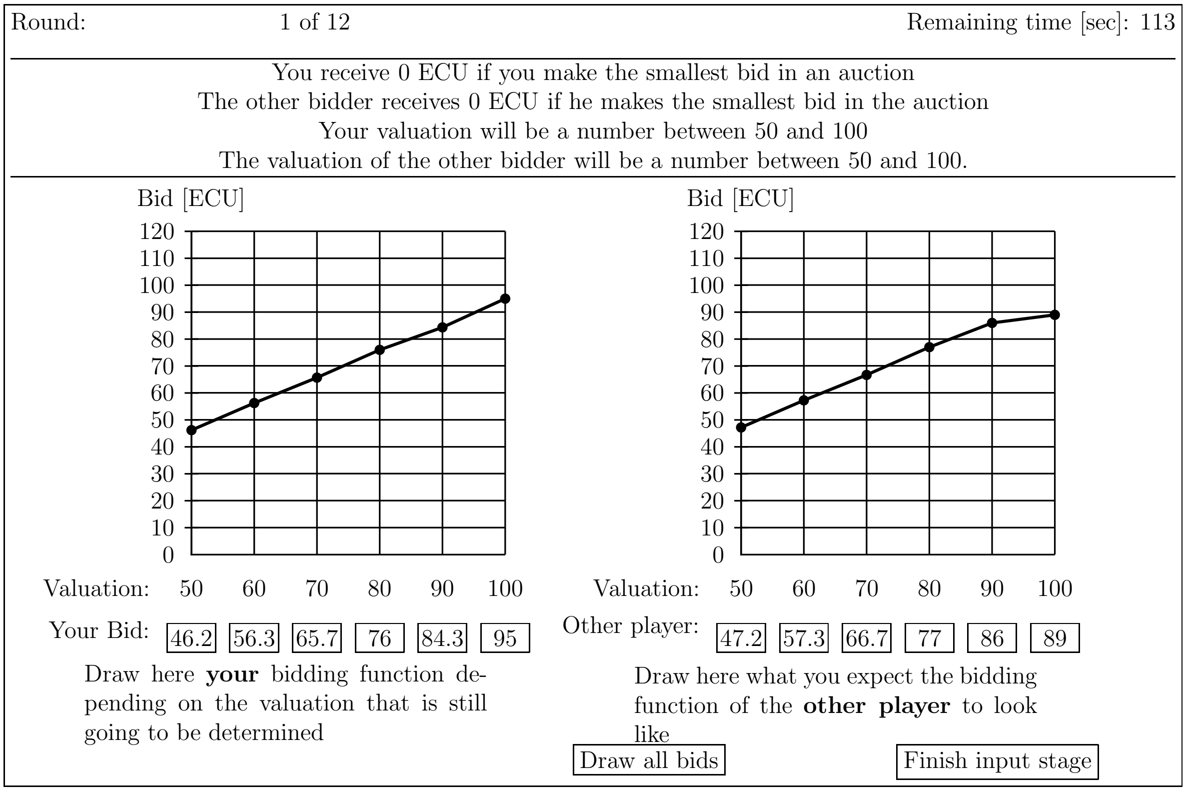


Figure 4: Stage 1: A typical input screen in the two 'expectations' treatments (translated into English)

and find that playing multiple auctions, indeed, induces bidders to behave in a more risk neutral way. They also find that already a small number of auctions played eliminates a substantial part of risk.[6] To keep things simple we rely on only five auctions in this experiment. A typical feedback screen is shown in figure 5.

Expectation feedback: In the expectation treatments players are informed about the precision of their expectations in the last stage of each round.

- In the baseline treatment 'no expectations' the last screen in each round only shows the total payoff of the current round.
- In the treatments 'expectation with info' and 'asymmetric' the last screen in each round is similar to the one shown in figure 6. A graph on the left shows the expected bid and, additionally, also the actual bid of the opponent. A small table on the right summarises the auction profit, the average difference between the expected bid and the actual bid, and the total payoff.

[6]Engelbrecht-Wiggans and Katok (2007b) run a different experiment where participants bid against computerised bidders instead of human opponents. In their setting elimination of risk does not have a significant impact on bidding behaviour.

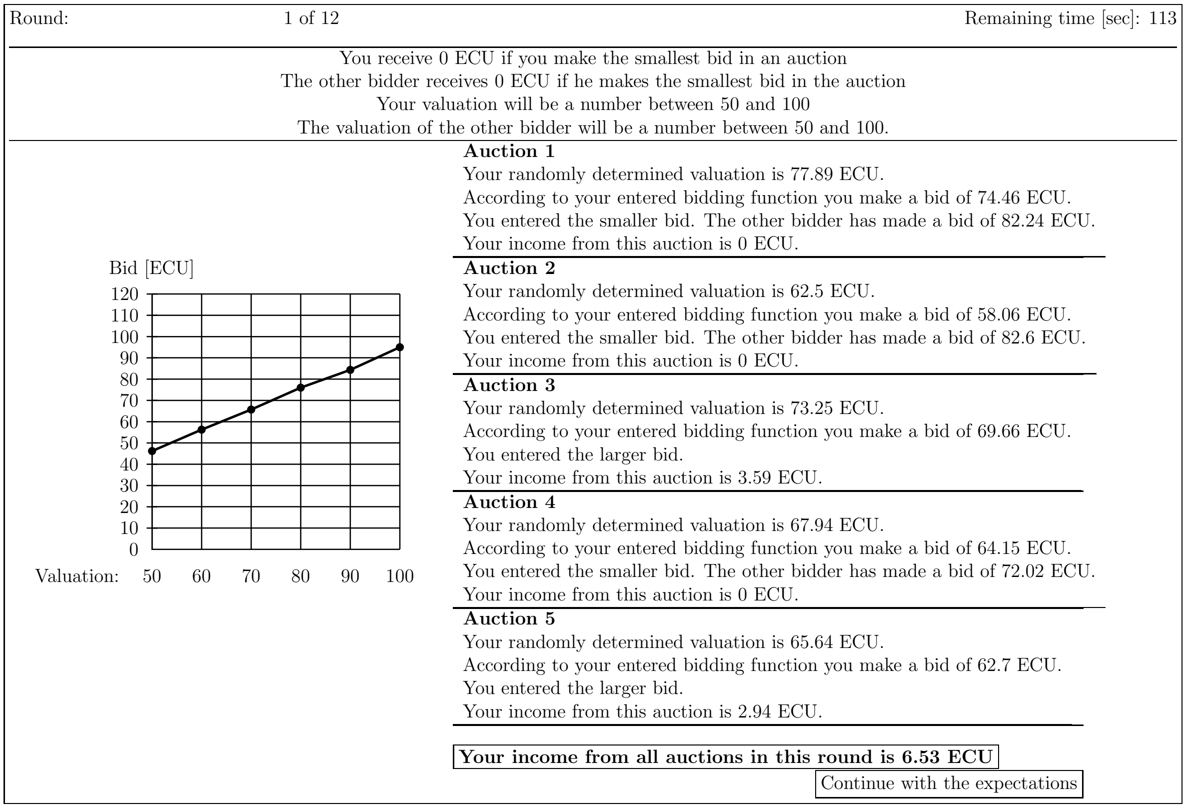


Figure 5: Stage 2: A typical feedback screen (translated into English)

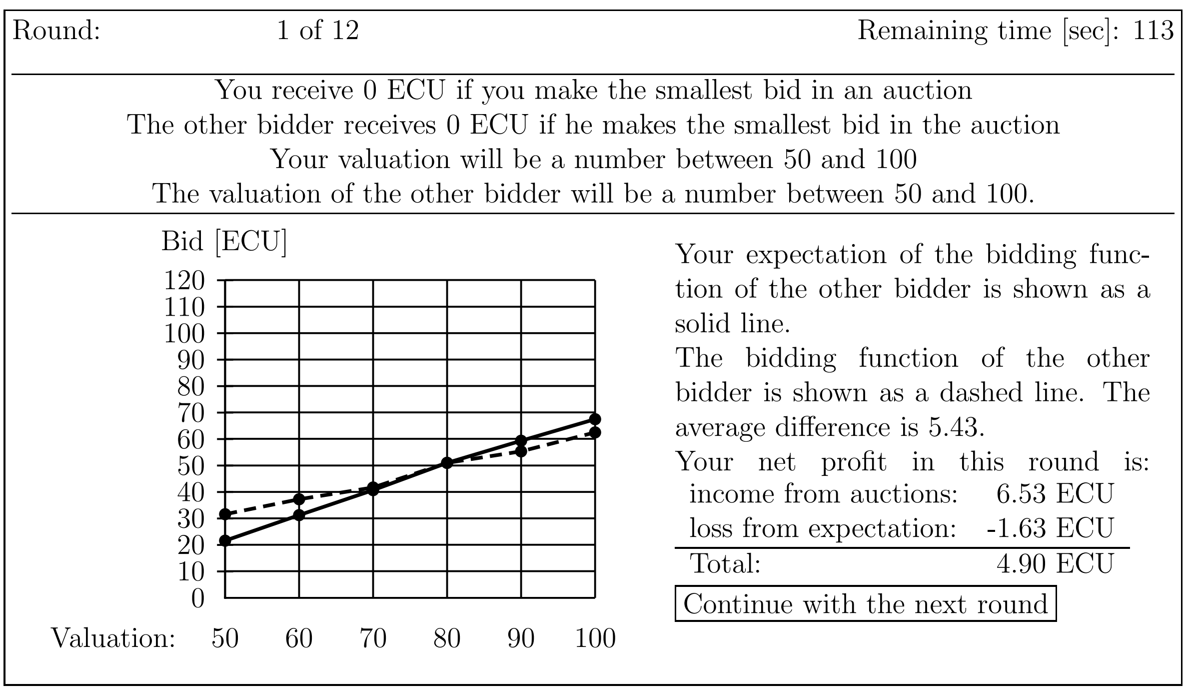


Figure 6: Stage 3: Expectation feedback in the expectation with info treatment

- In the 'expectation' treatment (without info) the only difference is that the graph on the left displays only the player's own expectation and not the actual bidding function of the opponent.

To incentivise participants' expectations we use the average of absolute differences between the actual bid of the opponent and the expected bid at the six points (symmetric treatments) or at the five or seven points (asymmetric treatments) where bids and expectations were made.

$$\delta^{\text{exp}} \equiv \frac{10}{\bar{x} - \underline{x} + 10} \sum_{x \in \{50,60,\ldots,\bar{x}\}} |b_x - b_x^e|$$

The average deviation δ^{exp} is multiplied by a conversion factor of 0.3 and then subtracted from the auction profit. The incentivisation implied by this rule is non-trivial; for the average participant, the total amount of income lost due to imprecise expectations is 12% of average total auction income. Given the widespread view that the degree of incentivisation reduces noise in the elicited data (Blanco et al., forthcoming, p. 3), supported by the findings[7] of Gächter and Renner (2010), we expect our expectations data to be non-arbitrary.

Point expectations and probabilistic expectations When introducing our method to elicit expectations, we implicitly assumed that individuals expect their opponents to use one specific bidding function b^{exp}. We call this a point expectation. More generally, a player might be uncertain about the specific bidding function of the opponent. This player might, e.g., expect to face an opponent with a bidding function b_1^{exp} with probability $\frac{1}{2}$ and to face an opponent with another bidding function b_2^{exp} again with probability $\frac{1}{2}$. A player might even have in mind an entire distribution over the space of all opponent's bidding functions. We call this a probabilistic expectation. Notice that a bidder with probabilistic expectations faces the same bid submission problem as a bidder with point expectations who plays against the average bidding function (where the averages are taken along the opponent's bids).

Since we are paying players according to their absolute deviations from the opponent's bidding function, players with probabilistic expectations should report as expectations in our experiment a least absolute deviation estimator, which is the median

[7] In a standard public goods game, Gächter and Renner (2010) compare expectations about the contribution behaviour of other participants elicited in an incentivised way to non-incentivised expectation data and find that incentivisation significantly increases the accuracy of expectations and that non-incentivised expectation data are not completely arbitrary.

expected bid.

Thus, as long as the difference between median and mean bidding functions is small, the expected bid reported by a payoff maximising bidder is close to the one that this bidder uses to calculate the best reply. To assess the order of magnitude of the problem at least approximately, let us assume that bidders apply the true distribution of bidding functions. Indeed, this distribution has a small negative skew. Medians are smaller than means by about 1.8% of the range of valuations (the size of the deviation does not depend much on the valuation). Thus, any deviation between reported expectations and bids of that magnitude is still perfectly rational.[8] We will, however, find that deviations are substantially larger.

Even if mean expected bids deviate substantially from median expected bids the incentive to hedge is small. The loss for reporting other than median expectations and optimising against other than mean expectations is large, and the profit from hedging is very low unless the distribution is extremely asymmetric and participants are very risk averse. Furthermore, Blanco et al. (forthcoming) systematically inquire into hedging effects in a belief elicitation experiment and find no behavioural cross-over effects from the elicitation task to the studied game. Hence, we do not expect hedging to be a problem. In the following we will disregard the problem of distributions of expectations and assume that bidders have point expectations of opponent's bidding functions.

4 Method and internal validity

Given the novelty of our experimental design we extensively check whether we actually measure what we intend to measure. Do participants understand the experiment, have they carefully thought about their expectations, and do they take their expectations into account when constructing their bids? To gain a first impression, figure 2 on page 6 shows some examples for bids and expectations observed in our experiments. In section 4.1 we check convergence of behaviour. Section 4.2 investigates treatment effects. In section 4.3 we examine whether participants in the experiments form reasonable expectations and section 4.4 explores whether bids follow actually best replies given these expectations. Only after all of these robustness checks are carried out with satisfying results we present our main findings in section 5.

[8] Note that the actual deviation of median and mean bidding functions for any participant forming probabilistic expectations can be either smaller or larger depending on the subjectively expected distribution of bidding functions that might deviate from the true one.

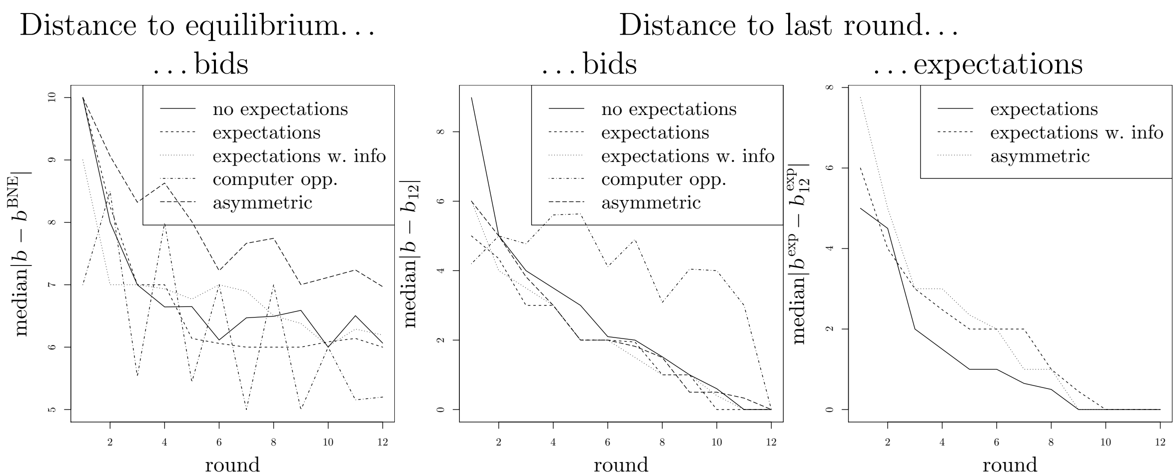


Figure 7: Convergence of bids and expectations

4.1 Convergence

The experiment is divided into 12 rounds. Figure 7 illustrates convergence of bids and expectations in the five different treatments.

A natural reference point for players' bids b are risk neutral Bayesian Nash equilibrium bids b^{BNE}. The left graph in Figure 7 shows the median of absolute equilibrium deviations $|b - b^{\mathrm{BNE}}|$ over the course of the experiment for all treatments. While the distance between experimental bids and equilibrium decreases during the first three or four rounds of the experiment it does not change very much during the second half of the experiment and remains considerably high, so that equilibrium deviations neither disappear over time nor substantially fluctuate. We take the stability of experimental bids as good news and an indication that, after a few initial rounds, players largely absorbed the experimental environment and the auction setting.

Another way to check for consistency and for stability of bids and expectations is to utilize their distances in any round of the experiment to their counterparts observed in the last round. Convergence requires the distance of a round's bid (expected bid) function to the one observed in the last round to decrease over the course of the experiment. The graph in the middle of Figure 7 shows the median distance of bids to their counterpart in the last round. Similarly, the graph on the right side does the same for expectations. We see that for all treatments, except the one with computerised opponents, bids and expectations approach the schedules submitted in the last round monotonically, providing further evidence for fairly consistent and stable behaviour in the second half of the experiment. The visibly slower convergence in the treatment with computerised opponents might hint that a situation with computerised opponents

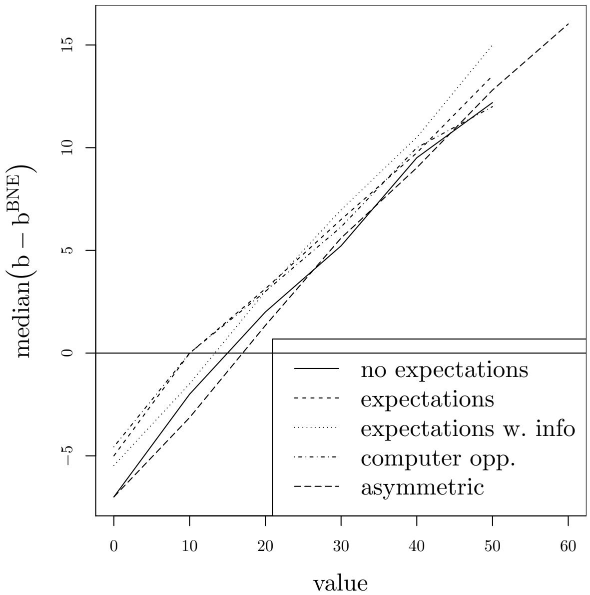


Figure 8: Median overbidding

is perhaps not as easy to understand and to play for participants in the lab as a treatment with human opponents.

4.2 Treatment effects

Do the different treatments we use affect bidding behaviour? Figure 8 compares median overbidding relative to the RNBNE under the five treatments. In equilibrium we should observe no overbidding, i.e. a horizontal line. The upward-sloping lines for the five treatments confirm a finding reported in many previous studies: there is overbidding for large valuations in all treatments.[9] We see that median bids are very similar in all five treatments. Overbidding is, if at all, even more pronounced in the symmetric treatments with expectation elicitation. To test this formally we estimate the following mixed effects model:

$$b_{ikt}(x) = \beta^* \cdot b^*(x) + \sum_T \beta^T \cdot d^T + \beta_0 + \nu_i + \nu_k + \epsilon_{iktx} \qquad (4)$$

where $b_{ikt}(x)$ is the bid of participant i in session k in round t for valuation x, $b^*(x)$ is the Bayesian Nash Equilibrium bid for valuation x, d^T is the treatment dummy where 'no expectations' is the baseline, ν_i is the random effect for the participant, ν_k is the random effect for the session, and ϵ_{iktx} is the residual. Table 2 presents estimation results. The

[9]Overbidding for the asymmetric case is documented in Güth et al. (2005) and Pezanis-Christou and Sadrieh (2003).

	β	σ	t	p value	95% conf	interval
(Intercept)	-7.32	0.581	-12.6	0.0000	-8.46	-6.18
β^*	1.7	0.00624	272	0.0000	1.69	1.71
expectations	1.84	0.908	2.03	0.0427	0.0609	3.62
expectations w. info	1.78	0.801	2.23	0.0260	0.213	3.35
computer opp.	1.43	1.24	1.15	0.2510	-1.01	3.87
asymmetric	0.132	0.889	0.148	0.8822	-1.61	1.87

Standard deviations, t-statistics, p-values, and confidence intervals are based on a parametric bootstrap with 1000 replications.

Table 2: Mixed effects estimation of equation 4

results confirm a small treatment effect on overbidding in the symmetric treatments 'expectations' and 'expectations with info'. Both treatment dummies are significant and indicate slightly more overbidding than is observed in the 'no expectations' treatment. Restricting the estimation of equation (4) to first round data only yields, however, insignificant treatment dummies. Accordingly, expectation elicitation does not affect bidding behaviour per se.

In addition to analysing overbidding relative to the RNBNE, our experimental design allows us to assess overbidding relative to participants' expectations regarding bidding behaviour of competitors.[10] To investigate overbidding relative to expectations in more detail, we estimate the following mixed effects model,

$$b_{ikt}(x) - b_{ikt}^{\exp}(x) = \beta_0 + \beta_x \cdot x + \sum_T \beta^T \cdot d^T + \nu_i + \nu_k + \epsilon_{iktx} \tag{5}$$

where the dependent variable is the difference between the bid and the expectation of participant i in session k in round t for valuation x, d^T is the treatment dummy where 'no expectations' is the baseline, ν_i is the random effect for the participant, ν_k is the random effect for the session, and ϵ_{iktx} is the residual. Table 3 presents estimation results separately for all periods, for the first, and for the second half of the experiment. We see there is overbidding of expectations for small values as indicated by the significantly positive intercept. But overbidding of expectations decreases in the value and turns negative for intermediate and large values so that participants underbid expectations for values not too small. There is no significant difference in overbidding of expectations when introducing feedback on expectations.

The behaviour of the weak bidder changes considerably during the experiment. While initially weak bidders clearly and significantly underbid, they learn, in line with

[10] Our thanks to an anonymous referee for pointing this out. Note that we explore overbidding relative to optimal bidding given participants' stated expectations in section 5.1 'Main Results'.

	all Periods	Period≤ 6	Period> 6
(Intercept)	0.935^{*}	1.098^{*}	0.772^{*}
	$[0.151; 1.719]$	$[0.157; 2.039]$	$[0.003; 1.541]$
β_x	-0.047^{***}	-0.035^{***}	-0.059^{***}
	$[-0.053; -0.041]$	$[-0.044; -0.027]$	$[-0.066; -0.052]$
with info	0.358	0.036	0.681
	$[-0.628; 1.345]$	$[-1.130; 1.203]$	$[-0.299; 1.660]$
asymmetric	0.362	-0.080	0.669
	$[-0.731; 1.455]$	$[-1.435; 1.275]$	$[-0.414; 1.751]$
weak	-0.184	-0.707^{*}	0.608^{*}
	$[-0.560; 0.192]$	$[-1.325; -0.089]$	$[0.110; 1.105]$
Deviance	110962.797	57018.379	52727.311
indep.obs.	27	27	27
participants	246	246	246
N	16872	8436	8436

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients.. Standard deviations, t-statistics, p-values, and confidence intervals are based on a parametric bootstrap with 1000 replications.

Table 3: Mixed effects estimation of equation (5)

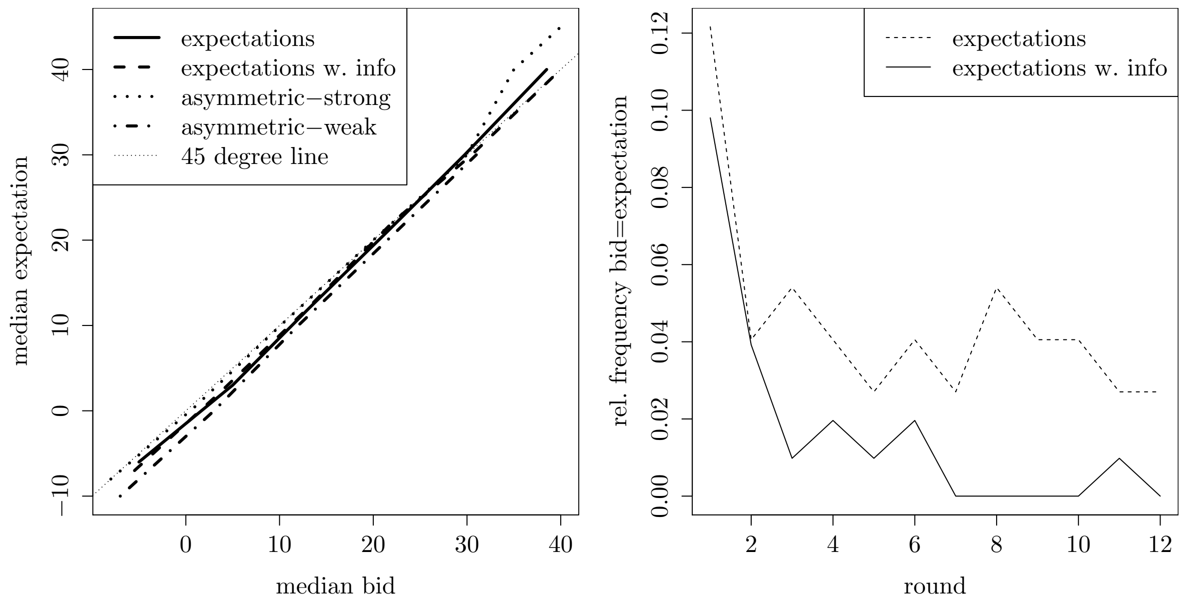
the equilibrium prediction, to bid more agressively than the expected bid of their strong opponent.

4.3 Quality of expectations

In our experiment subjects have an incentive to submit precise expectations. The larger the deviation of their expectation from their opponent's true bidding function, the smaller is their payoff. To analyse the quality of our participants' expectations we proceed in three steps: First, we show that expectations are, indeed, close to median bids. Second, we show that bidders actually learn. Their expectations react to changes in their opponents' bids. Third, we report how expectations respond to detailed feedback on the other bidder's bidding strategy.

The left graph in Figure 9 shows, for the different valuations, median bids and medians of expected bids for both expectation treatments. We see that, for all treatments, points are close to the 45° line, i.e. median expectations do not deviate much from median bids. To better understand individual heterogeneity we estimate individual expectations as a function of median bids $\bar{b}_t(x)$ for each round, treatment, and valuation. A participant who knows everything about bidding in this experiment, except the identity of her opponent should submit $\bar{b}_t(x)$ as expectations.[11]

[11]As above we did the same exercise with mean bids to obtain basically the same result. Since



The left graph shows median expectations over median bids. In a treatment where expectations would match bids exactly the graph would show five line segments on the 45 degrees line.
The right graph shows the relative frequency of bids equaling expectations. Large frequencies of bidders with the bids identical to expectations might suggest that bidders simply copy their bids to their expectations.

Figure 9: Bids and expectations

For each individual i in treatment k we estimate

$$b_{ikt}^{\exp}(x) = \beta_{ik}^{1}\bar{b}_t(x) + \beta_{ik}^{0} + u_{iktx}\,. \tag{6}$$

If expectations were always in line with actual bidding behaviour, we had $\beta_{ik}^1 = 1$ and $\beta_{ik}^0 = 0$. Figure 10 shows the estimated coefficients for each individual separately and 95% confidence ellipses for each treatment. Individual estimates of β^0 and β^1 are denoted with "+". The estimates' position of a hypothetical $L1$ bidder with expectations that assume $L0$ bidding of the competitor is marked with "$\circ$". The estimates' position of a hypothetical bidder with equilibrium expectations which coincide with bidding of $L1$, $L2$, and any higher level player in the symmetric case) is indicated by "$*$". "Correct" expectations which coincide with actual bidding, i.e. $\beta^0 = 0$ and $\beta^1 = 1$, are located at the intersection of the two dotted lines. We see that in all three treatments with expectation elicitation, bidders form, on average, neither naïve nor equilibrium expectations. Instead, expectations are, at least on average, consistent with actual bidding behaviour. A comparison of both symmetric expectations treatments, left and middle panel of figure 10, suggests that feedback on actual bidding by competitors

medians are less vulnerable to outliers we are concentrating on medians here.

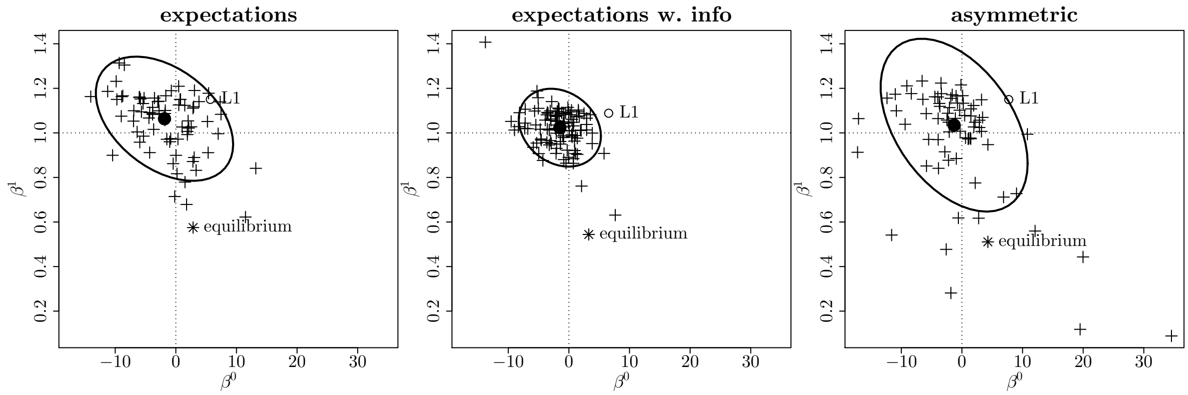


Figure 10: Individual estimates and 95% confidence ellipses for equation (6)

allows participants to form more precise expectations. In the asymmetric treatment it seems somewhat more difficult to form precise expectations.

Equation (6) allows us to assess the quality of expectations, but it does not reveal the causality between bids and expectations. A priori it is not obvious whether participants really have a good model of the behaviour of the population in mind or whether they use such a model to form reliable expectations. In the worst case participants might even follow the naïve procedure of copying the own bid into the expectation graph. But contrary to this possibility, almost none of the participants followed the copy-paste procedure. In the symmetric treatments 'expectations' and 'expectations with info' where direct copy-paste is possible, there are only 3.11% out of 2112 instances where the bidding schedule submitted coincides with the expectation schedule. The right graph in Figure 9 shows that the share of copy-paste observations starts with already small values at the beginning of the experiment and soon drops to negligibly small amounts.

To further inquire into the sophistication of bidders when forming expectations, we exploit that they are informed about the bidding schedule of the opponent in the treatments 'expectation with info' and 'asymmetric' and quantify the extent to which they use this information. Since bidders are matched in every round with a new random opponent, the bidding function of the opponent in the current round is not a perfect predictor for the opponent in the next round. Nevertheless, it provides new information about the distribution of bidding functions in the population. We use the opponent's bid as an explanatory variable for expectations and estimate the following equation in

	(7)	(8)
(Intercept)	0.315**	0.281***
	[0.126; 0.505]	[0.153; 0.410]
β^{other}	0.047***	0.055***
	[0.040; 0.053]	[0.047; 0.064]
β^{own}		0.282***
		[0.264; 0.300]
Deviance	63239.878	58725.368
indep.obs.	19	19
participants	172	172
N	9950	9620

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications. N is smaller for equation (8) since in the asymmetric treatment expectations are not always defined when bids are and vice versa.

Table 4: Mixed effects estimation of equations (7) and (8)

first differences[12] jointly for all treatments with feedback about opponents' bids:

$$\Delta_t b_{ikx}^{\text{exp}} = \beta^{\text{other}} \cdot \Delta_{t-1} b_{ikx}^{\text{other}} + \beta_0 + \nu_i + \nu_k + \epsilon_{iktx} \tag{7}$$

ν_i is the random effect for the participant, ν_k is the random effect for the session, and ϵ_{iktx} is the residual.

A bidder with no prior expectations who weights information from n rounds equally should have a β^{other} close to $1/n$. A bidder with strong prior expectations who is convinced that nothing new can be learned from the current opponent should have a $\beta^{\text{other}} = 0$. Estimation results are reported in the left column of table 4. Detailed results of treatment-specific estimations are provided in appendix C.

The coefficient of $\Delta_{t-1} b_{ikx}^{\text{other}}$ is positive and significantly different from zero. Changes in an opponent's individual bidding function seem to have an effect on a bidder's expectations for the next round.

Possibly a positive coefficient of $\Delta_{t-1} b_{ikx}^{\text{other}}$ in equation (7) arises due to an indirect effect where naïve bidders see opponents' bids rise, in response increase their own bids (without thinking about expectations), and then adjust their expectations in the same way they adjust their bids. To test this, we add $\Delta_t b_{ikx}^{\text{own}}$ as an explanatory variable in equation (8).

$$\Delta_t b_{ikx}^{\text{exp}} = \beta^{\text{other}} \cdot \Delta_{t-1} b_{ikx}^{\text{other}} + \beta^{\text{own}} \cdot \Delta_t b_{ikx}^{\text{own}} + \beta_0 + \nu_i + \nu_k + \epsilon_{iktx} \tag{8}$$

[12] Since b_{ikx}^{exp} and b_{ikx}^{other} are possibly correlated, we cannot use absolute values.

	β	σ	t	p value	95% conf	interval
(Intercept)	0.336	0.0805	4.17	0.0000	0.178	0.494
β^{own}	0.646	0.00804	80.4	0.0000	0.63	0.662
δ_0 (exp. w. info)	-0.1	0.111	-0.903	0.3665	-0.318	0.117
δ^{own} (exp. w. info)	-0.168	0.0138	-12.1	0.0000	-0.195	-0.141
β^{other} (exp. w. info)	0.0555	0.0051	10.9	0.0000	0.0455	0.0655

Standard deviations, t-statistics, p-values, and confidence intervals are based on a parametric bootstrap with 1000 replications.

Table 5: Mixed effects estimation of equation 9

Analogously to specification (7), a bidder with no prior expectations who weights information about opponents from n rounds equally should have a β^{other} close to $1/n$ and a β^{own} close to $1-1/n$.

Estimation results are shown in the right column of table 4. Detailed results for the different treatments are provided in appendix D. We see that, even if bidders are allowed to follow the naïve strategy outlined, the coefficient of $\Delta_{t-1}b^{\text{other}}_{ikx}$ is significantly positive, i.e. bids of opponents directly affect expectations.

We conclude our analysis of the quality of expectations by reporting how detailed feedback about other bidders' strategies influences expectations. To this end we compare expectations stated in the treatments 'expectations' and 'expectations with info' by estimating the following mixed-effects model:

$$\Delta_t b^{\text{exp}}_{ikx} = \beta_0 + \delta_0\, d^{\text{I}} + (\beta^{\text{own}} + \delta^{\text{own}}\, d^{\text{I}})\, \Delta_t b^{\text{own}}_{ikx} + \beta^{\text{other}}\, d^{\text{I}}\, \Delta_{t-1} b^{\text{other}}_{ikx} + \nu_i + \nu_k + \epsilon_{iktx} \quad (9)$$

where d^{I} is the treatment dummy indicating if the observation was obtained in the 'expectations with info' treatment. Table 5 reports regression results. The highly significant estimates of δ^{own} and β^{other} show that knowledge of competitors' bidding strategies directs participants' attention increasingly away from their own bidding behaviour towards the bidding behaviour of others when revising expectations. A comparison of expectations to median bids as illustrated by the left and the middle panel of figure 10 suggests that detailed feedback about bidding strategies used by other participants leads to more homogeneous expectations. To formally test if expectations are less heterogeneous in the 'expectations with info' treatment than in the 'expectations treatment', we inquire into the distance of expectations to median bids by estimating the following mixed-effects model

$$|b^{\text{exp}}_{ikt}(x) - \bar{b}_t(x)| = \beta_0 + d^{\text{I}} + \nu_i + \nu_k + \epsilon_{iktx} \quad (10)$$

	β	σ	t	p value	95% conf	interval
(Intercept)	5.36	0.367	14.6	0.0000	4.64	6.08
expectations w. info	-1.24	0.456	-2.72	0.0065	-2.14	-0.348

Standard deviations, t-statistics, p-values, and confidence intervals are based on a parametric bootstrap with 1000 replications.

Table 6: Mixed effects estimation of equation 10

where $b^{\text{exp}}_{ikt}(x)$ is the expectation of participant i in session k in round t for valuation x, $\bar{b}_t(x)$ is the median bid in round t and d^{I} is the treatment dummy indicating if the observation is from the 'expectations with info' treatment. The estimation results given in table 6 clearly confirm that detailed feedback about others' bidding strategies leads to more homogeneous expectations.

To summarise: We find that bidders in the experiment form expectations which are close to actual bids. These expectations follow to a significant amount the available information about actual opponents' bids. These findings are robust to dropping the assumption of symmetric bidders as they emerge in our 'asymmetric' treatment as well. Moreover, expectations are less heterogeneous if detailed feedback about competitors' bids is provided.

4.4 Quality of reactions to expectations

Independently of the quality of expectations, equilibrium bidding also assumes bidders to submit optimal bids given their expectations. To assess the descriptive power of this assumption, we construct for each bidder and each round the best reply given this bidder's expectations $b^{\text{exp}}(x)$. We call this best reply $b^{\text{opt}|\text{exp}}(x)$. Since in our experiment bids are stepwise linear we use a numerical procedure to find $b^{\text{opt}|\text{exp}}(x)$.[13] Some examples are shown in figure 2 on page 6.

If a rational bidder's expectations change, the bidder's best reply given expectations, $b^{\text{opt}|\text{exp}}$, responds optimally, and bids b move accordingly. To explore how consistent participants' bidding behaviour is with changes in best replies, we compare actual bids b with best replies $b^{\text{opt}|\text{exp}}$ and estimate the following equation in first differences for each participant i separately.

$$\Delta_t b_{iktx} = \beta^{\text{opt}|\text{exp}}_{\Delta} \cdot \Delta_t b^{\text{opt}|\text{exp}}_{iktx} + \beta_0 + u_{iktx} \,. \tag{11}$$

Results are shown in figure 11. Outliers have been eliminated using the BACON procedure (Billor et al., 2000).

[13]See appendix F for details on the numerical procedure.

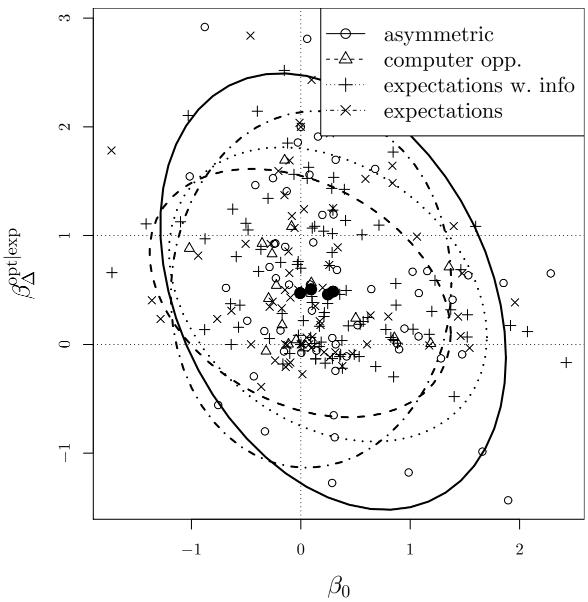


Figure 11: Individual estimates and 95% confidence ellipses for equation (11)

For a fully rational bidder we should find $\beta_\Delta^{\text{opt}|\text{exp}} = 1$. For a bidder who is slow in adapting and who also takes past experience into account, $0 < \beta_\Delta^{\text{opt}|\text{exp}} < 1$. Figure 11 shows that the degree to which bidders incorporate optimal responses to changes in expectations into their bidding behaviour varies considerably. Nevertheless we find $\beta_\Delta^{\text{opt}|\text{exp}} > 0$ for most bidders so that an expectation-driven change of the best reply triggers a change of bids into the same direction as that of the best reply. To test this more formally, we estimate the mixed-effects model specified by equation (12) for each treatment separately and also jointly.

$$\Delta_t b_{iktx} = \beta_\Delta^{\text{opt}|\text{exp}} \cdot \Delta_t b_{iktx}^{\text{opt}|\text{exp}} + \beta_0 + \nu_i + \nu_k + \epsilon_{iktx} \tag{12}$$

As before, ν_i is the random effect for the participant, ν_k is the random effect for the session, and ϵ_{iktx} is the residual. Results of the mixed effects estimation are shown in Table 7. We see that, indeed, for all treatments, except the 'expectations' treatment, confidence intervals for $\beta_\Delta^{\text{opt}|\text{exp}}$ are strictly between 0 and 1. Only in the 'expectations' treatment participants have a $\beta_\Delta^{\text{opt}|\text{exp}} > 1$, i.e. they are slightly overreacting to their best replies. Taken together, the reactions of bids in response to changed expectations are meaningful and follow the optimal response as given by the change of the best reply to a substantial amount.

	all	asymmetric	computer opp.	expectations	expectations w. info
(Intercept)	0.270***	0.735***	0.061	0.072	0.133
	[0.144; 0.397]	[0.338; 1.132]	[−0.645; 0.767]	[−0.157; 0.301]	[−0.031; 0.297]
$\Delta_t b^{\mathrm{opt}\vert\mathrm{exp}}$	0.659***	0.263***	0.177***	1.094***	0.930***
	[0.631; 0.686]	[0.195; 0.331]	[0.095; 0.259]	[1.052; 1.135]	[0.892; 0.967]
Deviance	117061.492	30902.458	8582.386	31728.315	42986.920
indep.obs.	44	8	17	8	11
participants	263	70	17	74	102
N	16958	4220	1122	4884	6732

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications.

Table 7: Mixed effects estimation of equation (12)

4.5 Experienced vs inexperienced subjects

According to the results of a post-experimental questionnaire, 102 out of the 379 subjects had acquired experience with experimental situations in general by having participated in other experiments prior to coming to our sessions. This raises the question if experience has substantially affected our results. To check for robustness on this issue, we included dummy variables that indicate experience and allowed for experience-related changes in intercepts and slopes in our regressions. We found that our main results essentially remain unaffected. Moreover, there were no significant differences in earnings. Nevertheless we found that the experienced subjects overbid slightly more in the baseline treatment 'no expectations' and that, in the treatments with information feedback about the bidding strategy of other subjects, expectations of the experienced bidders respond stronger to changes in the bids of others.

5 Results

In section 4 we have tested the reliability of our experimental framework. We have shown so far that bids and expectations are stable, that the different treatments we use to measure expectations affect bids only to a very small degree, that participants seem to make a reasonable effort to make good expectations, and that they try to incorporate these expectations into their bidding functions.

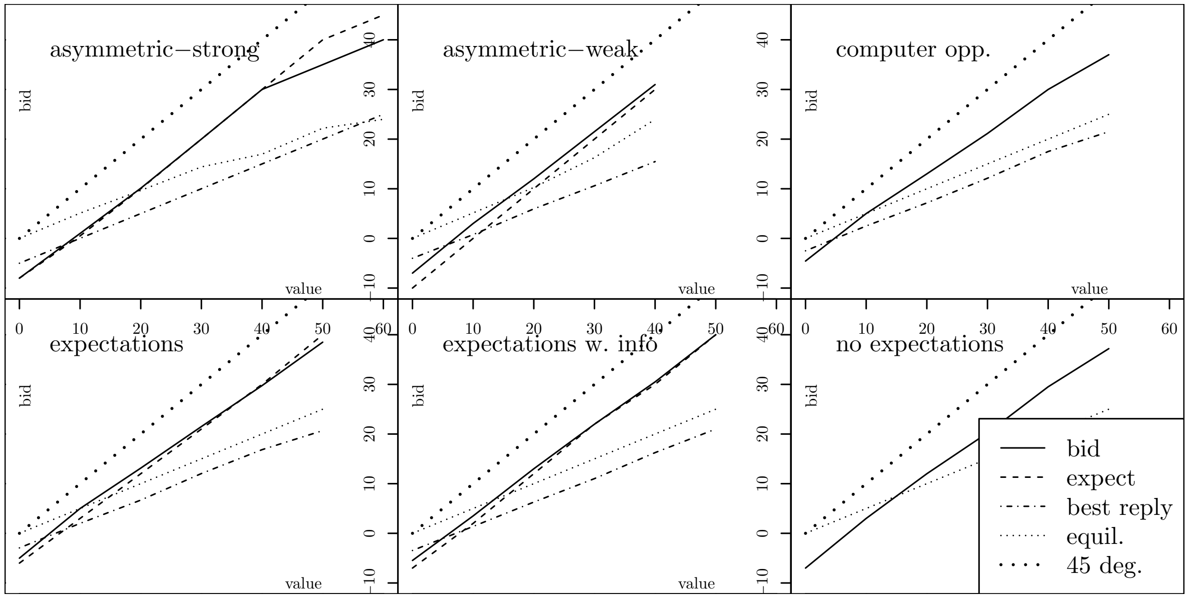
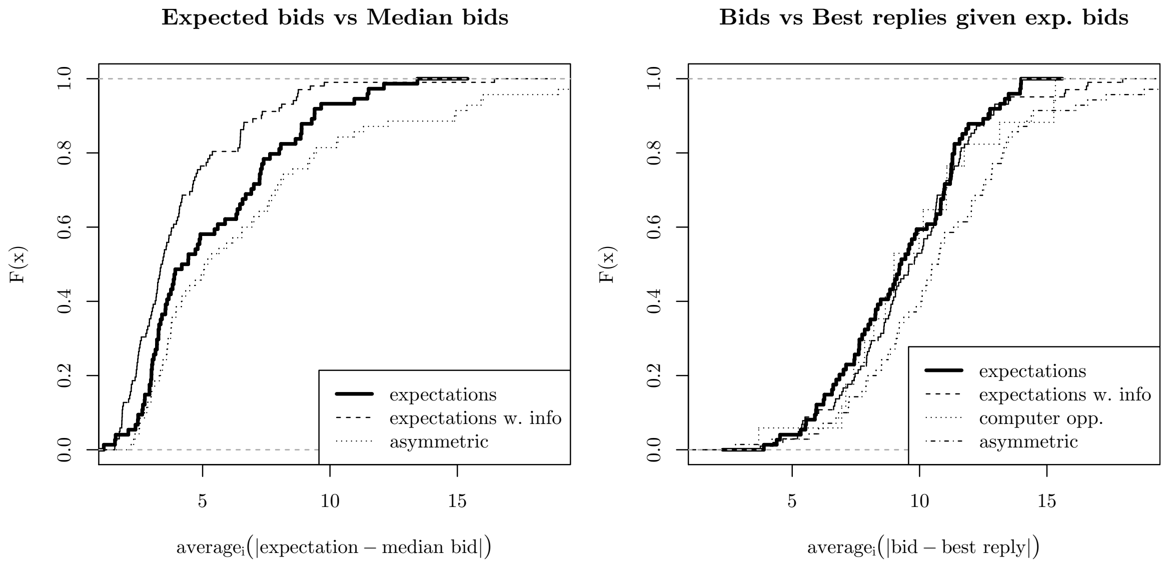


Figure 12: Median bids, expectations, and best replies

5.1 Main results

In this section we compare bids and expectations with Bayesian Nash equilibrium. As stated before, we do not aim at providing a complete and correct description of the thought process of real individuals. We are rather restricting ourselves to the derivation of Bayesian Nash equilibrium as our benchmark. In a first step we want to explore what happens on the way from the equilibrium bid $b^{\mathrm{BNE}}(x)$ to the best reply $b^{\mathrm{opt|exp}}$. In a second step we want to understand how the best reply $b^{\mathrm{opt|exp}}$ translates into the actual bid b. Taken together, we aim at understanding whether deviations between actual and equilibrium bids are rather due to non equilibrium expectations or whether they are rather due to deviations from risk-neutral best replies.

Figure 12 introduces the major variables that we will discuss in this section: Equilibrium bids, actual bids, actual expectations, and best replies to these expectations. For all of our treatments we find the same pattern: Bids are larger than equilibrium bids for high values and lower than equilibrium bids for low values. While expectations are rather close to median bids, see the left panel of figure 13, best replies to expectations are lower than equilibrium bids for all values; best replies also seem quite far away from actual bids for most values and all bidders as figures 12 and the right panel of figure 13 suggest.



The figure shows the empirical distribution functions of average (per participant) distances between expectations and median bids (left) and between actual bids and best replies to expectations (right). An average distance of 1 unit (ECU) can be interpreted as a parallel shift of the expectations or best reply schedule by one unit. A perfect fit between expectations and median bids or actual bids and best replies corresponds to an average distance of zero.

Figure 13: Distances of expectations to median bids (left) and bids to best replies given expectations (right)

We estimate the following two equations separately for each participant:

$$b_{ikt}^{\text{opt}|\text{exp}}(x) = \beta^{\text{BNE}} \cdot b_{ikt}^{\text{BNE}}(x) + \beta_0^{\text{BNE}} + u_{iktx} \tag{13}$$

$$b_{ikt}(x) = \beta^{\text{opt}|\text{exp}} \cdot b_{ikt}^{\text{opt}|\text{exp}}(x) + \beta_0^{\text{opt}|\text{exp}} + u_{iktx} \tag{14}$$

In equation (13) we use the best reply bid $b^{\text{opt}|\text{exp}}(x)$ as the dependent variable. If participants expect that their opponents use Bayesian Nash equilibrium bids, then $\beta^{\text{BNE}} = 1$ and $\beta_0^{\text{BNE}} = 0$. In equation (14) we regress the actual bid $b_i(x)$ on the best reply bid $b^{\text{opt}|\text{exp}}(x)$. If a player chooses always the best reply given the expected opponent's bid then $\beta^{\text{opt}|\text{exp}} = 1$ and $\beta_0^{\text{opt}|\text{exp}} = 0$. Figure 14 shows the distribution of the estimated coefficients.

Consider the coefficients estimated for equation (13). We see that for most participants $\beta^{\text{BNE}} \approx 1$ though intercepts β_0^{BNE} are clearly smaller than zero. The estimates of β^{BNE} and β_0^{BNE} are reflected in the median best reply in figure 12: The median of the best replies is almost parallel to the equilibrium bid, but located slightly below. In other words: Bidders seem to deviate in their expectations consistently from equilibrium bids. However, the deviation we find would rather explain underbidding, not overbidding. At

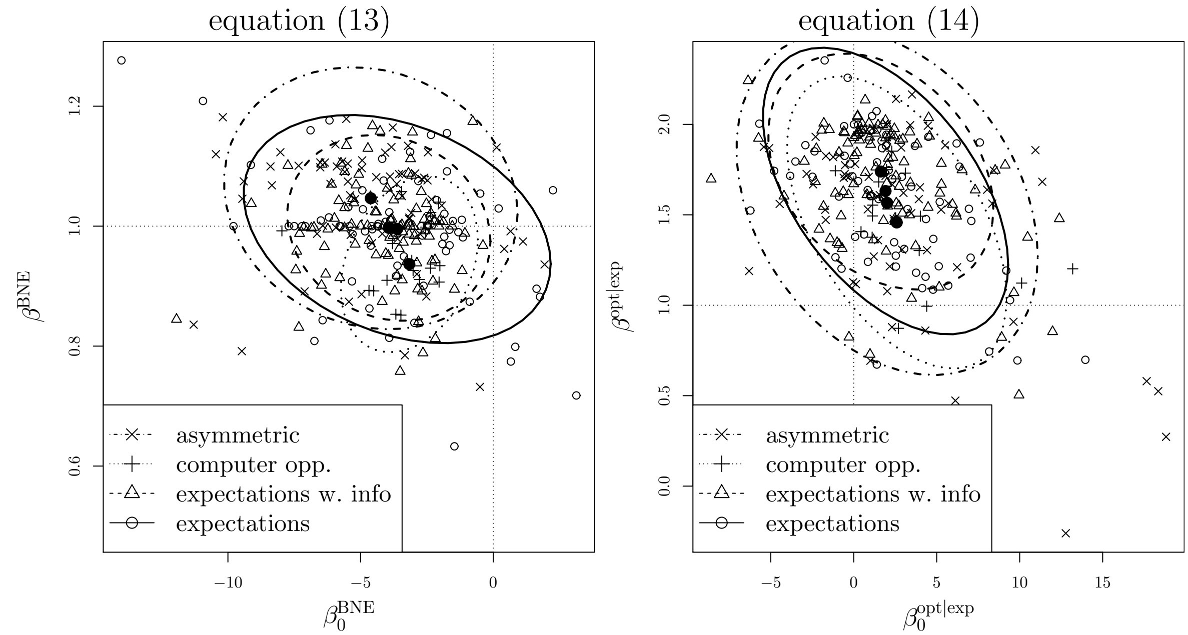


Figure 14: Individual estimates and 95% confidence ellipses for equations (13) and (14).

first sight this finding appears to be inconsistent with the regularity of overbidding as most experimental bids are over, and not under the equilibrium bids. The reason for the consistency of very low best replies $b^{\mathrm{opt|exp}}(x)$ relative to equilibrium bids b^{BNE} with the standard finding of overbidding emerges from the estimation results for equation (14) shown in the right part of figure 14. There it is easy to see that $\beta^{\mathrm{opt|exp}}$ is larger than one for most bidders. In figure 12 this reflects in the median bidding function that is steeper than the equilibrium bid.

To test this more formally, we estimate equations (15) and (16) separately for each treatment and also jointly:

$$b_{ikt}^{\mathrm{opt|exp}}(x) = \beta^{\mathrm{BNE}} \cdot b^{\mathrm{BNE}}(x) + \beta_0^{\mathrm{BNE}} + \nu_i + \nu_k + \epsilon_{iktx} \quad (15)$$
$$b_{ikt}(x) = \beta^{\mathrm{opt|exp}} \cdot b_{ikt}^{\mathrm{opt|exp}}(x) + \beta_0^{\mathrm{opt|exp}} + \nu_i + \nu_k + \epsilon_{iktx} \quad (16)$$

As before, ν_i is the random effect for the participant, ν_k is the random effect for the session, and ϵ_{iktx} is the residual. Results of the mixed effects estimation are shown in Tables 8 and 9. The estimation results confirm that β_0^{BNE} is significantly smaller than zero, but β^{BNE} is not significantly different from one. In other words, best replies to expectations are below, not above, the equilibrium bids. In table 9 we see that $\beta^{\mathrm{opt|exp}}$ is clearly larger than one.

Let us summarise: We find that there are two effects that determine bidding behaviour. First: Bidders' expectations are fairly accurate (see section 4.3). A best reply

	expectations	expectations w. info	computer opp.	asymmetric	all
β_0^{BNE}	-3.681^{***}	-4.126^{***}	-3.401^{***}	-4.741^{***}	-4.111^{***}
	$[-4.402; -2.961]$	$[-4.728; -3.523]$	$[-4.371; -2.431]$	$[-5.696; -3.787]$	$[-4.471; -3.751]$
β^{BNE}	0.990^{***}	0.994^{***}	0.943^{***}	1.018^{***}	0.996^{***}
	$[0.978; 1.002]$	$[0.985; 1.003]$	$[0.904; 0.982]$	$[1.004; 1.032]$	$[0.989; 1.002]$
Deviance	29915.997	39292.015	7825.298	29182.251	107075.939
indep.obs.	8	11	17	8	44
participants	74	102	17	70	263
N	5328	7344	1224	5040	18936

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications.

Table 8: Mixed effects estimation of equation (15)

	expectations	expectations w. info	computer opp.	asymmetric	all
$\beta_0^{\text{opt}\|\text{exp}}$	2.311^{***}	2.120^{***}	4.650^{***}	2.882^{***}	2.601^{***}
	$[1.379; 3.244]$	$[1.349; 2.892]$	$[2.907; 6.393]$	$[1.497; 4.266]$	$[2.121; 3.082]$
$\beta^{\text{opt}\|\text{exp}}$	1.552^{***}	1.638^{***}	1.277^{***}	1.396^{***}	1.521^{***}
	$[1.531; 1.572]$	$[1.620; 1.656]$	$[1.219; 1.336]$	$[1.360; 1.432]$	$[1.506; 1.536]$
Deviance	36182.928	50070.030	9338.025	39964.097	138483.758
indep.obs.	8	11	17	8	44
participants	74	102	17	70	263
N	5328	7344	1224	5040	18936

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications.

Table 9: Mixed effects estimation of equation (16)

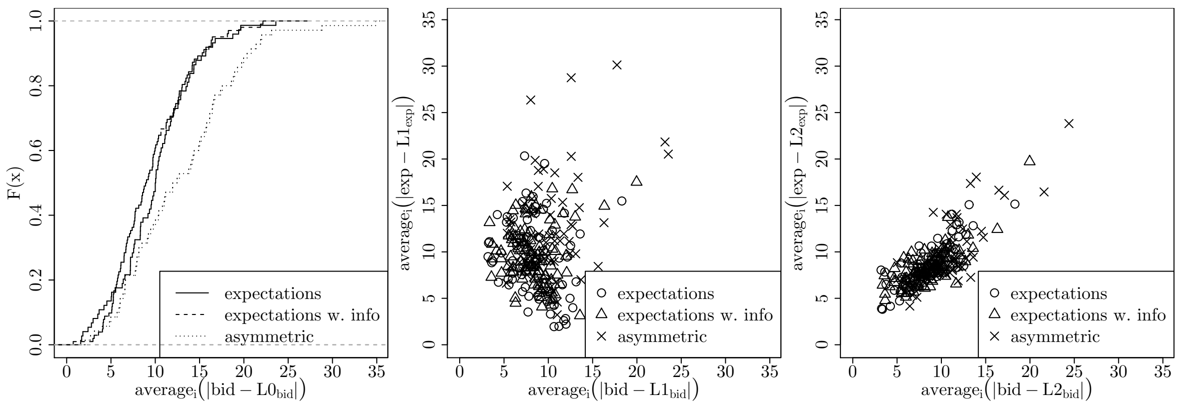


Figure 15: Distance of bids and expectations to level-k-bidders $L0$, $L1$ and $L2$

to these expectations leads to underbidding. Second: Bids are not best replies to bidders' expectations. This leads to overbidding. Since the second effect is stronger than the first one we observe that most bids exceed equilibrium bids in the end.

5.2 Level-k analysis of data

Here we relate our data to level-k reasoning. Note that our experimental design implies that any level-k-player with probabilistic expectations finds it optimal to state the median expectation.

First suppose that each level Lk expects to face a player of level $L(k-1)$ for sure. Our setting allows to distinguish between levels $L0$, $L1$, and $L2$ as described in section 2.4. To quantify the proximity of observed bidding schedules and/or expectation schedules relative to level-k predictions, we use averages (per participant) of the absolute difference between the predicted schedule and the observed one.

The panel in the middle of figure 15 indicates the average distance of bids (horizontal axis) and expectations (vertical axis) to $L1$ predictions. It is easy to see that there is no single instance of $L1$-predictions fitting perfectly; rather, the data show that average distances are quite large. The median of distances from bids to $L1$ is 8.54. This is almost as much as the average distance from $L0$ to $L1$ bids, which is 12.5. Hence, $L1$ does not seem to give a very precise description of bidding behaviour.

The situation for $L2$ is shown in the right part of figure 15. Analogously to $L1$, bids and expectations do not seem to be very well explained by $L2$. The median distance for bids is 8.93.

For $L0$ bidders, there is no prediction regarding expectations. The left panel of figure 15 shows empirical cumulative densities of the average distance of bids to $L0$ bids. Here the median is 9.99. Overall, the predictive power of $L0$, $L1$, or $L2$ appears similar and, in all cases, rather limited.

Now consider the iterated reasoning model with the assumption that level Lk_{mix} expects to face a distribution of lower levels such that the other player is possibly of any type $L(k-s)_{\text{mix}}$ where $s \in \{1, 2, \ldots, k\}$. First note that the left and the middle panel of figure 15 trivially apply in the same way to this variant of the level-k model since bids of $L0_{\text{mix}}$ and bids and expectations of $L1_{\text{mix}}$ are identical to those of their counterparts $L0$ and $L1$.

In the symmetric auction, any player of level $L2_{\text{mix}}$ or higher bids in the same way as $L1$ and $L1_{\text{mix}}$. Levels $L2+_{\text{mix}}$ only differ in the probability of believing to face $L0_{\text{mix}}$-bids that is denoted by λ. With our expectation elicitation rule, it is optimal to state $L0$-bids if $\lambda \geq 0.5$ and to state risk-neutral equilibrium bids (played by any level beyond $L0_{\text{mix}}$) if $\lambda \leq 0.5$. It follows that the data depicted in either the middle panel (for $\lambda \geq 0.5$) or that in the right panel (for $\lambda \leq 0.5$) of figure 15 apply to any level beyond $L1_{\text{mix}}$ for the symmetric auction. Similar reasoning applies to the asymmetric auction given that the belief of facing $L0$-bids by level strong $L2_{\text{mix}}$ is not unreasonably large, i.e. $\lambda < 2/3$. Hence the (modest) predictive power of the level-k model as outlined before is robust to allowing for best-replies to non-degenerate bid distributions of lower levels.

6 Concluding remarks

In this paper we investigate whether systematic deviations from equilibrium bidding behaviour in auctions are rather due to deviations from risk-neutral best replies that are based on correct expectations or whether they are rather due to wrong expectations. The first approach is quite standard and includes risk averse bidding behaviour, spite, regret, or quantal response mistakes. The second approach has only recently been employed by Eyster and Rabin (2005) and Crawford and Iriberri (2007).

Both explanations fit bidding behaviour in experiments. To distinguish between these explanations we propose an experiment where we can observe expectations and bids simultaneously. To keep things simple we use the context of a private value first-price sealed-bid auction.

Given the novelty of the approach we have carefully checked the internal validity of our setup. We have found that the expectations we measure are reliable, and that expectations also react to information in a reasonable way.

The main result was presented in section 5: Both approaches that we mentioned capture a part of the truth. Bidders make systematic mistakes in forming their expectations and in determining their strategy. However, we found that most of the deviations from equilibrium bids are not related to wrong expectations but to deviations from the risk-neutral best reply against these expectations.

Our results for first-price auctions complement, thus, the findings of Costa-Gomes and Weizsäcker (2008) for 3×3 games and of Offerman et al. (1996) for public good games: In both situations, ours and theirs, expectations resemble actual strategies fairly well. In both situations, however, strategies are not best replies to expectations. Furthermore, our results are consistent with experiments with computerised opponents, like Neugebauer and Selten (2006) or Charness and Levin (2009). These studies leave no room for expectation formation regarding any other player's strategy. Nevertheless, the authors still find overbidding, which suggests that failures of best replies with standard utility are also responsible for deviations from Bayesian Nash equilibrium. This message is in line with our conclusion following from a completely different research strategy.

Our results also support the standard approach to explain deviations from risk neutral Bayesian Nash equilibrium bids that requires bidding behaviour to be consistent with expectations about bidding behaviour. Risk aversion, regret (see Filiz-Ozbay and Ozbay, 2007), and spite (Morgan et al., 2003) are explanations that base on expectations which are correct. We can show that, indeed, the major part of the deviation from standard equilibrium is not due to wrong expectations regarding the strategies of other bidders but happens on the reply side.

Our first main result that bidders form expectations consistent with the bidding strategy of other bidders rules out that misperceived probabilities of winning the auction with some bid as implied by actual bidding behaviour (Armantier and Treich, 2009) can be rationalised by misperceived bidding strategies used by other bidders. Therefore, our finding that most equilibrium deviations are due to erroneous best replies is reinforced by the findings reported in Armantier and Treich (2009).[14] Our second main result of erroneous best replies to accurate expectations is a tentative one as the iden-

[14]Note that it is argued in Armantier and Treich (2009, p. 1097) that "... bidding behavior... may be best explained by a model in which subjects tend to best-respond to their... beliefs", but that they also find, but do not emphasize in their paper, that actual bidding behaviour is way off best-responses to stated probabilities as can be inferred from their figures 2 and 5. The reason why Armantier and Treich (2009) seem to advance the virtue of 'subjective' best-responses stems from their comparisons of various quantal response models. In their comparison, models with 'subjective' probabilities and risk-aversion are reported to fit the data better than a model with risk-neutrality but no 'subjective' probabilities. These comparisons, however, are relative statements about the performance of one subjective belief model while we entertain an absolute statement in our paper. Specifically, for any of the models considered by Armantier and Treich (2009), there is a gap between actual bids and the bid predictions of the model. The gap is reported to be smallest for a model with subjective probabilities as compared to the gaps associated with the remaining models. In strong contrast, in our paper, our statement of badly performing best responses to expectations is on the absolute size of the gap between actual bids and best responses, not on the gap size in comparison to gap sizes found with other models. Therefore, the conclusions of our paper and these reached by Armantier and Treich (2009) are consistent.

tification of its sources lies beyond the scope of the current paper. To this end, the studies Armantier and Treich (2009) and Dorsey and Razzolini (2003) suggest that one reason for erroneous best-replies is connected to the handling of probabilities, particulary when transforming the expected bidding strategies used by others (together with the underlying distribution of valuations) into the probability distribution of winning bids.

While Crawford and Iriberri's (2007) model of level-k thinking can explain bids or expectations when these are measured in isolation, we see that when we measure bids and expectations together their combination is not consistent with any level of k. Thus, our results are complementary to Ivanov et al. (2010) who experimentally study a second-price common-value auction and find that the Winner's Curse cannot be explained by models of best-response behaviour to inconsistent beliefs.

When we observe accurate expectations and inaccurate best replies in the lab we should keep in mind that forming precise expectations about opponents' bids might be easier in the lab than in real world auctions. Still, if the difference between bids and best replies is large in the lab we should expect this difference to be significant in the field as well.

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A Derivation of level-k reasoning

We begin by defining the two-step uniform distribution that proves useful for characterising optimal bidding in the level-k model. Then we state a lemma that proves useful when deriving the best reply functions of level-k. The two-step uniform distribution's definition nests the standard uniform distribution as a special case so that the lemma applies to uniformly distributed bids directly. To ease the application of the lemma to uniformly distributed bids, a corollary summarises the relevant results.

A.1 Auxiliaries

Definition 1 (Two-step uniform distribution)**.** *Let $g(b)$ denote a probability density function with support $[\underline{b}, \overline{b}] \subset \mathbb{R}$. Let $A \equiv [\underline{b}, \widehat{b}]$ and $B \equiv (\widehat{b}, \overline{b}]$ where $\underline{b} < \widehat{b} < \overline{b}$ and assume that density $g(b)$ is constant on either set, specifically, $g(b) = \delta_A > 0$ if $b \in A$ and $g(b) = \delta_B > 0$ if $b \in B$ such that $\delta_A \geq \delta_B$. The corresponding distribution function is*

$$G(b) = \begin{cases} 0 & \text{if } b < \underline{b}, \\ \delta_A \,(b - \underline{b}) & \text{if } b \in [\underline{b}, \widehat{b}], \\ \delta_A \,(\widehat{b} - \underline{b}) + \delta_B \,(b - \widehat{b}), & \text{if } b \in (\widehat{b}, \overline{b}], \\ 1 & \text{otherwise,} \end{cases} \tag{17}$$

where normalization implies $\delta_A \,(\widehat{b} - \underline{b}) + \delta_B \,(\overline{b} - \widehat{b}) \equiv 1$.
Remark: If the probability that a (two-step uniformly distributed) bid falling into interval A is $\lambda \geq 0$ so that the probability of the bid falling into interval B is $1 - \lambda$, then the constant densities are given by $\delta_A = \frac{1-\lambda}{\widehat{b}-\underline{b}} + \frac{\lambda}{\overline{b}-\underline{b}}$ and $\delta_B = \frac{\lambda}{\overline{b}-\underline{b}}$.

Lemma 1 (Best replies to two-step uniform bids)**.** *Let $G(b)$ be a two-step uniform distribution function and assume that bidder j submit bids in a first-price auction with two bidders according to $G(b)$. Then the (risk-neutral) best reply $b^*(x)$ of bidder i with value $x \geq \underline{b}$ facing the bid distribution of bidder j is the solution of the maximization problem* $\max_b \, G(b)\,(x - b)$ *and given by*

$$b^*(x) = \begin{cases} 0.5\,\underline{b} + 0.5\,x & \text{if } x \in [\underline{b}, \underline{x}_{\hat{b}}], \\ \widehat{b} & \text{if } x \in [\underline{x}_{\hat{b}}, \overline{x}_{\hat{b}}], \\ 0.5\,\underline{b} + 0.5\,x - \frac{\delta_A - \delta_B}{\delta_B}\,\frac{\widehat{b} - \underline{b}}{2} & \text{if } x \in [\overline{x}_{\hat{b}}, \overline{x}_{\overline{b}}], \\ \overline{b} & \text{if } x > \overline{x}_{\overline{b}}, \end{cases}$$

where $\underline{x}_{\hat{b}} = 2\widehat{b} - \underline{b}$, $\overline{x}_{\hat{b}} = \widehat{b} + \frac{\delta_A}{\delta_B}(\widehat{b} - \underline{b})$ and $\overline{x}_{\overline{b}} = 2\overline{b} + \frac{\delta_A}{\delta_B}(\widehat{b} - \underline{b}) - \widehat{b}$.

Proof. Omitted.

Sketch of Proof:
[This sketch of proof is intended for referees only and not for publication to economise on space.]

The first order condition of maximization problem $\max_b \, G(b)\,(x - b)$ is

$$G'(b^*)\,(x - b^*) = G(b^*)$$

and is necessary and sufficient for identifying a unique global maximum $\forall b^* \in$

$[\underline{b}, \overline{b}]\backslash\{\widehat{b}\}$.[15] Substituting the probability density and the probability distribution functions into the FOC yields

$$\begin{array}{lll} \delta_{\mathrm{A}}\,(x-b^*) & =\delta_{\mathrm{A}}\,(b^*-\underline{b}) & \text{if } b^*\in[\underline{b},\widehat{b}) \\ \delta_{\mathrm{B}}\,(x-b^*) & =\delta_{\mathrm{A}}\,(\widehat{b}-\underline{b})+\delta_{\mathrm{B}}\,(b^*-\widehat{b}) & \text{if } b^*\in(\widehat{b},\overline{b}] \end{array}$$

Solving both equations for b^* leads to the best-reply functions stated in the lemma on lines 1 and 3 where the boundaries are defined such that the domains of the FOC are taken care of. (By defining the value of x that leads to a best reply of $\widehat{b}$ in either case.) If value x is sufficiently high, i.e. $x > \overline{x}_{\overline{b}}$, so that the FOC cannot be satisfied on $[\underline{b}, \overline{b}]$ (then the best-reply function on line 3 of the lemma implies a best-reply exceeding the competitor's largest bid $\overline{b}$), there is a boundary solution such that the best reply is given by $\overline{b}$ as stated on line 4 of the lemma.

It remains to show the result stated on line 2 of the lemma. For $\delta_{\mathrm{A}} = \delta_{\mathrm{B}}$ distribution $G(b)$ reduces to a standard uniform distribution so that line 2 of the lemma is redundant (=nested by line 1). Henceforth, let $\delta_{\mathrm{A}} > \delta_{\mathrm{B}}$. It is straightforward that $\forall x \in (\underline{x}_{\hat{b}}, \overline{x}_{\hat{b}})$, the FOC cannot be satisfied. This is due to the fact that the LHS is discontinuous at $\widehat{b}$, specifically, it jumps downward from $\delta_{\mathrm{A}}(x-\widehat{b})$ to $\delta_{\mathrm{B}}(x-\widehat{b})$, and that the LHS is always strictly larger than the RHS for $b < \widehat{b}$ but strictly smaller for $b > \widehat{b}$. As a result, the best reply is $\widehat{b}$ for all $x \in (\underline{x}_{\hat{b}}, \overline{x}_{\hat{b}})$. (On line 2 of the lemma, the interval is closed to emphasize that the best-reply is continuous.) □

Corollary 1. *Let the competitor's bids be uniformly distributed on support $[\underline{b}, \overline{b}]$ and let the probability density be given by $\delta = \frac{1}{\overline{b}-\underline{b}}$. This bid distribution is a special case of two-step uniform distributions with $\delta_A = \delta_B = \delta$. The application of Lemma 1 implies the best reply function for uniformly distributed bids that is given by:*

$$b^*(x) = \begin{cases} 0.5\,\underline{b} + 0.5\,x & \text{if } x \in [\underline{b}, 2\overline{b} - \underline{b}], \\ \overline{b} & \text{if } x > 2\overline{b} - \underline{b}. \end{cases}$$

A.2 Derivation when level-k plays level-$(k-1)$ only

A.2.1 The symmetric auction

In the symmetric auction, the value distribution is U$[50, 100]$ for any bidder. Let us start from the anchor level $L0$ that bids truthfully, i.e., $b_{L0}(x) = x$, implying uniformly

[15] For this note that the LHS of the FOC is monotonically decreasing from some strictly positive number (depending on the density) to $-\infty$ while the RHS of the FOC is monotonically increasing from 0 to 1 as b monotonically increases from $\underline{b}$ to ∞. (Note that the FOC is, strictly speaking, not differentiable at $\underline{b}$ and $\overline{b}$, however, we use then its right-hand limit or left-hand limit respectively.)

distributed $L0$-bids on $[50, 100]$. $L1$ expects this bidding behavior, $b^{\text{exp}}_{L1}(x) = b_{L0}(x)$, and by corollary 1 the best reply is $b_{L1}(x) = 25 + 0.5x$ which coincides with RNBNE bidding. Accordingly, $L2$ expects equilibrium bids, $b^{\text{exp}}_{L1}(x) = b_{L1}(x)$, that are uniformly distributed on $[50, 75]$. By corollary 1 the best reply is $b_{L2}(x) = 25 + 0.5x$. Analogously $L3$ and any higher level form the same expectation as $L2$ and bid in the same way.

A.2.2 The asymmetric auction

In the asymmetric auction, the weak bidder's value distribution is U$[50, 90]$ while the strong bidder's one is U$[50, 110]$. In the asymmetric setting there are two versions of any level, a weak one and a strong one, since a bidder knows whether she is weak or strong. Here, the truthfully bidding anchor levels bid according to $b_{wL0}(x) = b_{sL0}(x) = x$ so that $wL0$-bids are uniformly distributed on $[50, 90]$ while $sL0$-bids are uniformly distributed on $[50, 110]$.

Weak $L1$ expects his competitor to be $sL0$ and by corollary 1 his best reply is $b_{wL1}(x) = 25+0.5x$ where bids are uniformly distributed on $[50, 70]$. Similarly $b_{sL1}(x) = 25 + 0.5x$ so that bids are uniformly distributed on $[50, 80]$.

Weak $L2$ expects his competitor to be $sL1$ and by corollary 1 his best reply is $b_{wL2}(x) = 25 + 0.5x$ where bids are uniformly distributed on $[50, 70]$ again.

For strong $L2$, however, the best reply is more involved since $sL2$ expects $wL1$ submitting uniformly distributed bids on $[50, 70]$ and never submits a bid exceeding the largest bid of $wL1$, i.e. 70, so that corollary 1 implies

$$b_{sL2}(x) = \begin{cases} 25 + 0.5x & \text{if } x \in [50, 90], \\ 70 & \text{if } x \in [90, 110]. \end{cases}$$

Weak $L3$ expects that $sL2$'s bid is 70 with probability 1/3 and that the remaining probability is uniformly spread over $[50, 70]$, hence, the cumulative bid distribution function is $B^{\text{exp}}_{wL3}(b) = (b - 50)/30$ for $b \in [50, 70)$, $B^{\text{exp}}_{wL3}(b = 70) = 5/6$ due to fair tie-breaking and $B^{\text{exp}}_{wL3}(b > 70) = 1$. Except of both jumps in probability at $b = 70$, the maximisation problem is the same as that of, e.g., $wL2$. Here, however, we have to verify if the expected payoff of that solution is a global maximum or if it is dominated by a bid of $70 + \epsilon$. Indeed, for all $x > 110 - \sqrt{1200} \approx 75.359$, $wL3$ prefers to win the auction for sure with a bid of $70 + \epsilon$ $(\epsilon \to 0)$.[16] Since there always exists a smaller $\epsilon > 0$ that allows to increase the expected payoff, the payoff-maximizing bid is undefined for

[16] If it is optimal to submit a bid smaller than 70, then the best reply is $b^o = 25 + 0.5x$ implying expected payoff $(b^o - 50)/30\,(x - b^o) = (0.5x - 25)^2/30$. This falls short of the certain payoff of $x - (70 + \epsilon)$ with $\epsilon \to 0$ for $x > 110 - \sqrt{1200}$.

any $x \in (110 - \sqrt{1200}, 90]$. It follows that the best replies of strong $L4$, weak $L5$, etc., are not defined.

Strong $L3$ faces the same problem as strong $L2$, hence, $b_{sL3}(x) = b_{sL2}(x)$ with probability mass concentrating at a bid of 70 so that the best reply of weak $L4$ and beyond are undefined, too.

A.3 Derivation when level-k plays a distribution of lower levels

When assuming that a player of level Lk plays against a distribution of lower levels, the best replies of levels $L0$ and $L1$ coincide with their counterparts playing against $L(k-1)$ only. The lowest level that plays against a non-degenerated distribution of lower levels is $L2$ expecting to face a competitor of either level $L0$ or $L1$.

For specifying the distribution of lower levels, we impose the consistency condition of the cognitive hierarchy model implying that the probability of facing any lower level $L(k-s)$, $s \in 1, \ldots, k$, as expected by Lk, coincides with the relative population share of this level as prevailing in the population of players conditional on facing lower levels only.[17] In particular we denote the population share of level Lk by $\lambda_{Lk} \geq 0$ ($\sum_{k=0}^{\infty} \lambda_{Lk} = 1$) and the probability that a player of level Lk believes to face any lower level Ls ($s \in \{0, \ldots, k-1\}$) by $\lambda_{Lk}^{Ls} = \frac{\lambda_{Ls}}{\sum_{j=0}^{k-1} \lambda_{Lj}}$.

A.3.1 Levels $L2$ and higher in the symmetric auction

The probability that $L2$ expects to face an $L0$-player is $\lambda_{L2}^{L0} > 0$, bidding uniformly on $[50, 100]$, and the probability of facing an $L1$-player is $1 - \lambda_{L2}^{L0}$, bidding uniformly on $[50, 75]$. For simplicity, we suppress the index of λ in the following derivations and reintroduce it where it matters. The bid distribution that $L2$ expects is two-step uniform with $\widehat{b} = 75$, $\delta_{\mathrm{A}} = (2-\lambda)/50$, $\delta_{\mathrm{B}} = \lambda/50$ and follows from definition 1 as

$$B_{L2\mathrm{mix}}^{\exp}(b) = \begin{cases} \frac{2-\lambda}{50}\,(b - 50) & \text{if } b \in [50, 75], \\ 1 - 2\lambda + \frac{\lambda}{50}\,b & \text{if } b \in (75, 100]. \end{cases}$$

By lemma 1, the best reply of $L2$ to the distribution of $L0$- and $L1$-bidders is

$$b_{L2\mathrm{mix}}(x) = \ 25 + 0.5x \quad (x \in [50, 100]),$$

where $L2$ submits uniformly distributed bids on $[50, 75]$ as does $L1$.

[17] See Camerer et al. (2004), p.864f.

By the consistency condition of beliefs, the probability that any level higher than $L2$ expects to face $L0$ does not exceed the probability of $L2$ believing to face $L0$, i.e. $\lambda_{L2}^{L0} \geq \lambda_{Lk}^{L0}$ $(k \in \{3, \dots, \infty\})$. Therefore, any level higher than $L2$ also finds it optimal to bid according to the best reply function of $L2$ (which is that of $L1$). Levels $L1$ and higher differ only in the probability of believing to face $L0$ bids that, with increasing level, converges from one (believed by $L1$) to the true population share λ_{L0} (believed latest by $L\infty$) monotonically.

A.3.2 Levels $L2$ and higher in the asymmetric auction

Let the probability that weak $L2$ expects to face a strong $L0$-player be $\lambda_{wL2}^{sL0} > 0$, bidding uniformly on $[50, 110]$, hence, the probability of facing a strong $L1$-player is $1 - \lambda_{wL2}^{sL0}$, bidding uniformly on $[50, 80]$. With suppressing the index, the distribution of bids that $wL2$ expects is two-step uniform with $\widehat{b} = 80$, $\delta_{\mathrm{A}} = (2-\lambda)/60$, $\delta_{\mathrm{B}} = \lambda/60$ and follows from definition 1 as

$$B_{wL2\mathrm{mix}}^{\mathrm{exp}}(b) = \begin{cases} \frac{2-\lambda}{60}\,(b-50) & \text{if } b \in [50, 80], \\ 1 - \frac{11}{6}\lambda + \frac{\lambda}{60}\,b & \text{if } b \in (80, 110]. \end{cases}$$

By lemma 1, the best reply of $wL2$ to the distribution of $sL0$- and $sL1$-bidders is

$$b_{wL2\mathrm{mix}}(x) = \ 25 + 0.5x \quad (x \in [50, 90]),$$

where $wL2$ submits uniformly distributed bids on $[50, 70]$ as does $wL1$.

Strong $L2$ expects $wL0$ (bidding uniformly on $[50, 90]$) with probability λ_{sL2}^{wL0} and $wL1$ (bidding uniformly on $[50, 70]$) with probability $1-\lambda_{sL2}^{wL0}$. Thus, the bid distribution expected by $sL2$ is two-step uniform with $\widehat{b} = 70$, $\delta_{\mathrm{A}} = (2-\lambda)/40$, $\delta_{\mathrm{B}} = \lambda/40$ and follows from definition 1 as

$$B_{sL2\mathrm{mix}}^{\mathrm{exp}}(b) = \begin{cases} \frac{2-\lambda}{40}\,(b-50) & \text{if } b \in [50, 70], \\ 1 - \frac{9}{4}\lambda + \frac{\lambda}{40}\,b & \text{if } b \in (70, 90]. \end{cases}$$

By lemma 1, the best reply of $sL2$ to the distribution of $wL0$- and $wL1$-bidders is

$$b_{sL2\mathrm{mix}}(x) = \begin{cases} 25 + 0.5x & \text{if } x \in [50, 90], \\ 70 & \text{if } x \in [90, x_\lambda], \\ 45 - \frac{20}{\lambda} + 0.5x & \text{if } x \in [x_\lambda, 110], \end{cases}$$

where $x_\lambda = 70 + \frac{2-\lambda}{\lambda} \cdot 20 > 90$ for $\lambda < 1$.

It follows that $sL2$'s best reply is flat on an interval such that $sL2$ bids 70 with strictly positive probability unless believing that the population consists of $L0$ players only ($\lambda = 1$). Hence, $wL3$ believes that a bid of 70 is submitted with strictly positive probability. It follows that the best reply of $wL3$ to $sL2$ is not defined for some values analogously to the situation of $wL3$ playing against $sL2$ with certainty. Hence, there are no best replies for $wL3, sL4, \ldots$ The best reply of $sL3$, however, exists and coincides with that of $sL2$, but, for the same reasons as stated for wL, there are no best replies for $wL4, sL5, \ldots$, too.

B List of independent observations

date	treatment	place	participants
20091102-1559-0	asymmetric	Jena	10
20091102-1559-1	asymmetric	Jena	8
20091110-1347-0	asymmetric	Jena	10
20091110-1347-1	asymmetric	Jena	8
20091112-1349-0	asymmetric	Jena	8
20091112-1349-1	asymmetric	Jena	8
20091112-1600-0	asymmetric	Jena	10
20091112-1600-1	asymmetric	Jena	8
20091110-1555	computer opp.	Jena	17
20050511-10:51-0	expectations	Magdeburg	10
20050511-10:51-1	expectations	Magdeburg	10
20050511-14:55-0	expectations	Magdeburg	10
20050511-14:55-1	expectations	Magdeburg	10
20050512-09:01-0	expectations	Magdeburg	10
20050512-09:01-1	expectations	Magdeburg	8
20050512-12:59-0	expectations	Magdeburg	8
20050512-12:59-1	expectations	Magdeburg	8
20050207-10:53-0	expectations w. info	Mannheim	8
20050209-14:09-0	expectations w. info	Mannheim	12
20050209-16:11-0	expectations w. info	Mannheim	6
20050414-10:37-0	expectations w. info	Magdeburg	10
20050414-10:37-1	expectations w. info	Magdeburg	10
20050414-16:35-0	expectations w. info	Magdeburg	10
20050414-16:35-1	expectations w. info	Magdeburg	10
20050415-08:59-0	expectations w. info	Magdeburg	8
20050415-08:59-1	expectations w. info	Magdeburg	8
20050415-11:11-0	expectations w. info	Magdeburg	10
20050415-11:11-1	expectations w. info	Magdeburg	10
20031211-18:23-0	no expectations	Mannheim	14

20031212-10:45-0	no expectations	Mannheim	14
20040517-12:21-0	no expectations	Mannheim	8
20040517-12:21-1	no expectations	Mannheim	6
20040517-17:17-0	no expectations	Mannheim	8
20040517-17:17-1	no expectations	Mannheim	8
20040519-15:53-0	no expectations	Mannheim	8
20040519-15:53-1	no expectations	Mannheim	10
20050414-08:55-0	no expectations	Magdeburg	10
20050414-08:55-1	no expectations	Magdeburg	10
20050414-13:17-0	no expectations	Magdeburg	10
20050414-13:17-1	no expectations	Magdeburg	10

Table 10: List of all sessions

C Detailed estimation results for equation (7)

	all	asymmetric	expectations w. info
(Intercept)	0.315**	0.206	0.414**
	[0.118; 0.512]	[−0.147; 0.558]	[0.128; 0.699]
β^{other}	0.047***	0.034***	0.079***
	[0.040; 0.054]	[0.024; 0.044]	[0.069; 0.090]
Deviance	63239.878	25817.629	36712.601
indep.obs.	19	8	11
participants	172	70	102
N	9950	3830	6120

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications.

D Detailed estimation results for equation (8)

	all	asymmetric	expectations w. info
(Intercept)	0.281***	0.304*	0.236*
	[0.152; 0.410]	[0.071; 0.537]	[0.051; 0.420]
β^{other}	0.055***	0.055***	0.056***
	[0.046; 0.064]	[0.039; 0.071]	[0.046; 0.065]
β^{own}	0.282***	0.105***	0.478***
	[0.264; 0.300]	[0.076; 0.134]	[0.457; 0.499]
Deviance	58725.368	22618.006	35003.196
indep.obs.	19	8	11
participants	172	70	102
N	9620	3500	6120

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 · 0.1. 95% confidence intervals are shown in square brackets below coefficients. Confidence intervals and p-values are based on a parametric bootstrap with 1000 replications.

E Conducting the experiment

Participants were recruited by email and could register for the experiment on the internet.

- At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats in the laboratory.
- Then participants took a simple language test (participants had to find the correct word or form to complete a sentence). Those who failed the language test on at least two items out of ten could not participate (this did not happen very often since participants knew about the language test when they booked the experiment).
- The remaining participants obtained written instructions in German (see section E.1). These instructions vary slightly depending on the treatment. In the following we give a translation of the instructions.
- After answering control questions on the screen (see section E.2) subjects entered the treatment. After completing the treatment they answered a short questionnaire on the screen and were paid in cash. The experiment was done with z-Tree Version 3α (the final version is documented in Fischbacher, 2007).

E.1 Instructions

General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can—depending on your decision—gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. **During the experiment no communication is permitted.** Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to exclusion from the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU (Experimental Currency Unit). Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and paid to you in **cash**. The conversion rate will be shown on your screen at the beginning of the experiment.

Information regarding the experiment

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct **12 rounds**. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are **two bidders**. In each round you will be allocated randomly to another participant for the auction. **Your co-bidder in the auction changes in every round.** The bidder with the highest bid has obtained the object. If bids are the same the object will be allocated randomly.

For the auctioned object you have a valuation in ECU. This valuation lies between 50 and 100 ECU and is determined randomly in each round. **From this range you will obtain in each round new and random valuations for the object.** The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from 50 to 100 from which also your valuations are drawn. All valuations between 50 and 100 are equally probable. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. **You will not know the valuation of the other bidder.**

Experimental procedure

The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

1st Stage

In the first stage of the experiment you see the following screen *[[here the instructions show a screen similar to figure 3 or figure 4. Other than the figure the screenshots in the instructions did not provide an example bidding function.]]*

At that stage **you do not know your own valuation for the object in this round.** On the left side[18] of the screen you are asked to enter a bid for **six hypothetical valuations** that you might have for the object. These six hypothetical valuations are 50, 60, 70, 80, 90, and 100 ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on "draw bids". In the graph the hypothetical valuations are shown on the horizontal axis, the bids are shown on the vertical axis. Your input in the table is shown as six points in the diagram. **Neighbouring points**

[18] In the no expectation treatment this was the right side.

are connected with a line automatically. These lines determine your bids for all valuations **between** the six valuations for which you have entered a bid.

[[the following paragraph is only shown in the treatments with expectations: On the right side you are asked to enter **your expectations regarding the bids of the other bidder**. Please enter again for six hypothetical valuations **your expectation of the bid of the other bidder**. If your expectation regarding the bids of the other bidder deviates from the actual bids of the other bidder then an amount which depends on the size of the deviation will be subtracted from your account.*]]*

The screen of the other bidder looks identical. He also enters bids for six hypothetical valuations *[[the following only in treatments with expectations:* and expectations regarding your bids*]]*. You and the other bidder cannot see your mutual bids and expectations.

2nd Stage

The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but **five auctions**. This is done as follows: **Five times a random valuation is determined** that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen:

[[here the instructions show a screen similar to figure 5. Other than these figures the screenshots in the instructions do not provide example bidding functions, bids, valuations, and payoffs.]]

For each of your five valuations the computer determines your bid according to the graph from stage 1. If a valuation is precisely 50, 60, 70, 80, 90, or 100 then the computer takes the bid that you gave for this valuation. If a valuation is between these points then your bid is determined according to the connecting line. In the same way the bids of the other bidder are determined for his five valuations. Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.

Your income from the auction:

For each of the five auctions the following holds:

- The bidder with the higher bid gets the valuation he had for the object in this auction added to his account minus his bid for the object.
- The bidder with the smaller bid gets **no income** from this auction.

[[[the next two paragraphs and the screenshot are only shown in the treatments with expectations:

The possible reduction if expectations are not correct The following screen again shows the expectations you entered in the first stage:

[[here the instructions show a screen similar to figure 5 or 6. Other than these figures the screenshots in the instructions do not provide examples for expected bidding functions, no examples for income and no examples for a loss.]]

The average difference between your expectations and the actual bids of the other bidder for the six hypothetical valuations 50, 60, 70, 80, and 100, multiplied with the conversion factor that is shown on the screen, is subtracted from your account.*]]]*

You total income in a round is the sum of the ECU income from those auctions in this round *[[the following part is only shown in the treatments with expectations:* **minus the reduction for your incorrect expectations regarding the other bidder.***]]*

This ends one round of the experiment and you see in the next round again the input screen from stage 1.

At the end of the experiment your total ECU income from all rounds will be converted into Euro and paid to you in cash together with your Show-Up Fee of 3.00 Euro.

Please raise your hand if you have questions.

E.2 Control questions

After participants had read the instructions they were asked to answer control questions. These questions were implemented with z-Tree. Questions were presented and answered sequentially. When a question was answered correctly, participants saw the text "This answer is correct" (in German). Otherwise participants saw the text "This answer is not correct". In this case they got a brief explanation how to derive the correct answer for this question.

The structure of this treatment was (translated into English) as follows:

- The following control questions are supposed to improve your understanding of the experiment. We use some arbitrarily chosen examples to make you familiar with the calculation of profits and other rules in the auction.

  Please answer the following questions. You can check yourselves whether your answers are correct. The actual experiment will start after the last question.

- Please note: When you enter numbers with a decimal fraction you have to use the decimal point as a separator, not the decimal comma.

- If you need a calculator, please click on the symbol on your screen.

1. Assume your valuation is 63.25 ECU and your bid that is derived from the bid function in the graph is 40 ECU. What is your income in this auction if
   (a) the other bidder bids less than your bid?
   (b) the other bidder bids more than your bid?
2. Assume your valuation is 50 ECU and your bid that is derived from the bid function in the graph is 60 ECU. What is your income in this auction if
   (a) the other bidder bids less than your bid?
   (b) the other bidder bids more than your bid?
3. Assume your valuation in this auction is 76.20 ECU. What is your valuation in the next auction?
   - 76.20 ECU / one cannot say / 0 ECU
4. Assume your valuation in this auction is 51.67 ECU. What is the valuation of the other bidder in this auction?
   - one cannot say / 51.67 ECU / 100 ECU
5. The following table shows an example for your expectations regarding the bids of the other bidder as well as the actual bids of the other bidder.

value	expected bid	actual bid
50	40	40
60	40	40
70	40	30
80	40	40
90	40	50
100	40	50

   What amount will be subtracted from your account due to wrong expectations if the conversion factor is 1?
6. Assume that in one round you have won one auction with a valuation of 80 ECU and a bid of 62 ECU. Furthermore, you lost 7 ECU due to wrong expectations. What is your total income from this round?

E.3 End of the experiment

At the end of the experiment participants completed a questionnaire, again with z-Tree. From their answers we know that about 20% of all participants were female, their median age was 23, about 68% were students of economics and business administration, 73% had participated already in another experiment, and 33% already in another experiment with auctions (Participants could attend only one of the treatments we describe in this paper). They found the experiment not very complicated (on a scale from 1 (not complicated) to 5 (very complicated) the average rating was 1.56).

After participants had completed the questionnaire each of them obtained a sealed envelope with their profit from the experiment and left the laboratory.

F Calculating best replies

We call the vector of own valuations `myVal` and the vector of opponent's valuations `otherVal`. The vector of expected bids of the opponent is called `expect`. We start our search at the equilibrium bidding function `myEqBid`. The best reply is found by `R`'s optimisation function:

```
> optRes <- optim(par = myEqBid, fn = function(x) -payMat(merge(mmVals(x,
+     myVal), monoBid(expect, otherVal)), sum = TRUE),
+     method = "BFGS")
```

whith the following functions:

```
> payMat

function(bothBids,sum=FALSE) {
  xx <- within(bothBids,{
    lcritObid <- pmin(pmax(minObid,minBid),maxBid)
    rcritObid <- pmax(pmin(maxObid,maxBid),minBid)
    rAval <- lval+(maxBid-lbid)*(rval-lval)/(rbid-lbid)
    rCval <- lval+(rcritObid-lbid)*(rval-lval)/(rbid-lbid)
    lCval <- lval+(lcritObid-lbid)*(rval-lval)/(rbid-lbid)
    minAval <- pmin(rAval,rCval)
    maxAval <- pmax(rAval,rCval)
    minCval <- pmin(rCval,lCval)
    maxCval <- pmax(rCval,lCval)
    cPay <- ifelse(maxObid==minObid,0,(1 / (rval - lval) * (rbid - lbid) * width / (maxObid - minObi
    aPay <- (width * (1 - 1 / (rval - lval) * (rbid - lbid)) * (maxAval ^ 2 - minAval ^ 2)) / 0.2e1
  })
  if(sum) with(xx,sum(cPay)+sum(aPay))
  else xx
}

> mmVals
```

```
function(bid,val=c(50,60,70,80,90,100)) {
  lbid<-bid
  rbid<-c(bid[-1],NA)
  lval<-val
  rval<-c(val[-1],NA)
  minBid<-pmin(lbid,rbid)
  maxBid<-pmax(lbid,rbid)
  as.data.frame(cbind(lbid,rbid,lval,rval,minBid,maxBid))[-length(bid),]
}
```

```
> monoBid
```

```
function(bid,val=c(50,60,70,80,90,100)) {
  width<-c(val[-1],NA)-val
  lbid<-bid
  rbid<-c(bid[-1],NA)
  minObid<-pmin(lbid,rbid)
  maxObid<-pmax(lbid,rbid)
  sbids <- sort(unique(bid))
  nbids <- length(sbids)
  xx <- as.data.frame(cbind(width,minObid,maxObid))[-length(bid),]
  if (nbids>1) ox <- merge(xx,cbind(minB = sbids[-nbids],maxB = sbids[-1]),all=TRUE)
  else ox <- merge(xx,cbind(minB = sbids,maxB = sbids),all=TRUE)
  ox <- within(ox,share<-ifelse(maxObid==minObid,1,(maxB-minB)/(maxObid-minObid)) * width * (minB>=m
  ox <- with(ox,aggregate(ox$share,list(minObid=minB,maxObid=maxB),sum))
  names(ox)[3]<-c("width")
  ox
}
```

G Statistical calculations are done in R

- R version 2.12.0 (2010-10-15), `i386-pc-mingw32`
- Base packages: base, datasets, graphics, grDevices, grid, methods, splines, stats, utils
- Other packages: cacheSweave 0.4-5, car 2.0-6, filehash 2.1-1, lattice 0.19-13, lme4 0.999375-37, MASS 7.3-8, Matrix 0.999375-44, memisc 0.95-31, nnet 7.3-1, robustbase 0.5-0-1, robustX 1.1-2, stashR 0.3-3, survival 2.35-8, xtable 1.5-6
- Loaded via a namespace (and not attached): digest 0.4.2, nlme 3.1-97, stats4 2.12.0, tools 2.12.0

H Referee's Appendix: Differences between the experienced and the inexperienced subjects

[This appendix is not intended for publication.] The reason why we do not cover effects regarding subjects prior participation in auction experiments is that our results remain essentially unaffected by this type of experience. Importantly, our main results are robust to differentiating between both experience levels of subjects. We find, however, that the experienced subjects slightly overbid more in the treatment 'no expectations' and that, in the treatments with information feedback about the bidding strategy of other subjects, expectations of the experienced bidders respond stronger to changes in the bids of others. To give you the possibility to verify our assessments that the main results remain unaffected and to check the results when we find significant effects, we provide the relevant data analysis below. We did not include the data analysis in the paper due to considerations of space.

1. **Overview and insignificant income differences**
   A total of 102 subjects reported that they previously participated in auction experiments. These subjects will be referred to as 'experienced subjects'. The average profit of an experienced subject was 10.64€ with a standard deviation of 3.75€ as compared to non-experienced subjects with average income of 10.37€ and standard deviation 4.97€ overall. The number of experienced subjects varies across treatments considerably: only six experienced subjects participated in the 'asymmetric' treatment and only one experienced subject participated in the 'computerized opponents' treatment, while there were around 30 experienced subjects in every other treatment. Although we find regression results based on only one or six subjects unreliable, we report them for the sake of completeness. To explore

	independent observations	participants
no expectations	10	27
expectations	8	35
expectations w. info	11	33
computer opp.	1	1
asymmetric	6	6
all	36	102

Table 11: Overview of treatments with experienced subjects

   if there is any significant income difference due to experience, we estimate the following mixed effects model

$$\pi_{ik} = \beta_0 + \sum_T \beta^T \cdot d^T + \sum_T \beta^T_X \cdot d^T_{\mathrm{X}} + \nu_k + \epsilon_{ik} \tag{18}$$

   where $\pi_{ikt}(x)$ is the total income of participant i in session k, d^T is the treatment dummy where 'no expectations' is the baseline and d^T_{X} is the treatment dummy indicating an extra income due to experience in treatment T; ν_k is the random effect for the session and ϵ_{ik} is the residual. Table 12 reports estimation results and shows that the experienced subjects did not earn any significant extra income. To

	β	σ	t	p value	95% conf	interval
(Intercept)	326	19.3	16.9	0.0000	288	364
asymmetric	2.45	32.2	0.0762	0.9393	-60.8	65.7
computer opp.	63.1	38.7	1.63	0.1043	-13.1	139
expectations	-43.6	32.3	-1.35	0.1782	-107	20
expectations w. info	-27.5	28.1	-0.978	0.3288	-82.8	27.8
d_{X}asymmetric	10.8	56.4	0.192	0.8482	-100	122
d_{X}computer opp.	-45.9	144	-0.318	0.7505	-330	238
d_{X}expectations	51.6	30.1	1.72	0.0871	-7.56	111
d_{X}expectations w. info	33	28.6	1.15	0.2493	-23.2	89.2
d_{X}no expectations	-20.6	29.7	-0.693	0.4889	-78.9	37.8

Standard deviations, t-statistics, p-values, and confidence intervals are based on a parametric bootstrap with 1000 replications. (We omit this statement in all following tables that provide regression results in this response letter.)

Table 12: Mixed effects estimation of equation (19)

explore if there is any significant income difference due to experience, we estimate the following mixed effects model

$$\pi_{ik} = \sum_T \beta^T \cdot d^T + \sum_T \beta_X^T \cdot d_{\mathrm{X}}^T + \nu_k + \epsilon_{ik} \tag{19}$$

where $\pi_{ikt}(x)$ is the total income of participant i in session k, d^T is the treatment dummy where 'no expectations' is the baseline and d_{X}^T is the treatment dummy indicating an extra income due to experience in treatment T; ν_k is the random effect for the session and ϵ_{ik} is the residual. Table 12 reports estimation results and shows that the experienced subjects did not earn any significant extra income.

2. **Main results robust to distinguishing experience levels of subjects:**

To test if the presence of experienced subjects affects our main results, we augment regression specifications (15) and (16) by dummy variables indicating prior experience. Specifically, we introduce a dummy that allows for a shift in the intercept and another dummy that allows for a different slope for the experienced bidders.

$$\begin{aligned} b_{ikt}^{\mathrm{opt|exp}}(x) &= \beta^{\mathrm{BNE}} \cdot b^{\mathrm{BNE}}(x) + \beta_0^{\mathrm{BNE}} + \beta^{\mathrm{BNE}}{}_{\mathrm{X}} d_{\mathrm{X}} b^{\mathrm{BNE}}(x) + \\ &\quad + \beta_0^{\mathrm{BNE}}{}_{\mathrm{X}} d_{\mathrm{X}} + \nu_i + \nu_k + \epsilon_{iktx} && (20) \\ b_{ikt}(x) &= \beta^{\mathrm{opt|exp}} \cdot b_{ikt}^{\mathrm{opt|exp}}(x) + \beta_0^{\mathrm{opt|exp}} + \beta^{\mathrm{opt|exp}}{}_{\mathrm{X}} d_{\mathrm{X}} b_{ikt}^{\mathrm{opt|exp}}(x) + \\ &\quad + \beta_0^{\mathrm{opt|exp}}{}_{\mathrm{X}} d_{\mathrm{X}} + \nu_i + \nu_k + \epsilon_{iktx} && (21) \end{aligned}$$

The results of the mixed effects estimation of equations (20) and (21) are given in tables 13 and 14.

	expectations	expectations w. info	computer opp.	asymmetric	all
β_0^{BNE}	-3.165^{***}	-4.281^{***}	-3.422^{***}	-4.697^{***}	-4.112^{***}
	$[-3.975; -2.355]$	$[-5.054; -3.508]$	$[-4.473; -2.370]$	$[-5.686; -3.709]$	$[-4.483; -3.741]$
β^{BNE}	0.982^{***}	1.003^{***}	0.943^{***}	1.009^{***}	0.995^{***}
	$[0.965; 0.999]$	$[0.992; 1.014]$	$[0.902; 0.984]$	$[0.994; 1.024]$	$[0.987; 1.004]$
$\beta_0^{BNE}{}_X$	-1.091	0.482	0.352	-0.511	0.003
	$[-2.150; -0.032]$	$[-0.309; 1.272]$	$[-3.853; 4.557]$	$[-2.763; 1.742]$	$[-0.597; 0.603]$
$\beta^{BNE}{}_X$	0.018	-0.029^{**}	-0.000	0.099^{***}	0.001
	$[-0.006; 0.042]$	$[-0.049; -0.008]$	$[-0.166; 0.166]$	$[0.050; 0.149]$	$[-0.014; 0.015]$
Deviance	29912.023	39283.855	7825.348	29166.958	107075.934
indep.obs.	8	11	17	8	44
participants	74	102	17	70	263
N	5328	7344	1224	5040	18936

Table 13: Mixed effects estimation of equation (20)

First, consider the results reported in table 13 where best replies to held expectations are regressed on the equilibrium bid and the experience dummies. Clearly, experience does not shift the intercept, the estimated coefficient $\beta_0^{BNE}{}_X$ of the intercept dummy d_X is insignificant in all treatments. For $\beta^{BNE}{}_X$ we find significant estimates in the 'asymmetric' and in the 'expectations with info' treatments where, importantly, the magnitude is quite small as compared to the estimated β^{BNE} for the inexperienced subjects. Further, when conducting a robustness check by reestimating with data only from the second half of the experiment (rounds 7-12), the positive $\beta^{BNE}{}_X$ in the 'asymmetric' treatment becomes insignificant, while the $\beta^{BNE}{}_X$ in the 'expectations with info' treatment remains significant and negative of similar size. The estimates of the slopes are $\beta^{BNE} = 1.003$ for the inexperienced bidders and $\beta^{BNE} + \beta^{BNE}{}_X = 0.974$ for the experienced bidders, so that we find a small effect here that is significant: The experienced bidders in the 'experience with info' treatment form expectations such that their best replies are a little bid closer to the equilibrium bids.

Second, consider the results reported in table 14 where actual bids are regressed on best replies given held expectations and experience dummies. None of the estimates of the intercept dummies on experience ($\beta_0^{opt|exp}{}_X$) is significant. The coefficient $\beta^{opt|exp}{}_X$ is estimated significantly for all treatments except the 'expectations' treatment; in any case, since the number of experienced subjects in the 'asymmetric' treatment (6 subjects) and in the 'computer opponents' treatment (1 subject) is small, and, hence, results are not too reliable, we disregard these two estimates. For the dummy on differences in slope in the 'expectations with info' treatment, we find a highly significant negative estimate that is robust to reestimation with data only from the second half of the experiment. The estimates for the slope are $\beta^{opt|exp} = 1.675$ for the inexperienced subjects and

	expectations	expectations w. info	computer opp.	asymmetric	all
$\beta_0^{\text{opt}\|\text{exp}}$	2.302***	2.115***	4.976***	2.932***	2.745***
	[1.173; 3.431]	[1.187; 3.042]	[3.182; 6.770]	[1.554; 4.311]	[2.163; 3.327]
$\beta^{\text{opt}\|\text{exp}}$	1.551***	1.675***	1.251***	1.373***	1.506***
	[1.524; 1.579]	[1.654; 1.697]	[1.188; 1.314]	[1.333; 1.412]	[1.490; 1.522]
$\beta_0^{\text{opt}\|\text{exp}}{}_{\text{X}}$	0.019	0.091	−6.089	−0.724	−0.534
	[−1.436; 1.474]	[−1.426; 1.607]	[−13.550; 1.372]	[−5.388; 3.939]	[−1.625; 0.558]
$\beta^{\text{opt}\|\text{exp}}{}_{\text{X}}$	0.001	−0.123***	0.492***	0.259***	0.056***
	[−0.040; 0.042]	[−0.162; −0.085]	[0.220; 0.764]	[0.128; 0.389]	[0.024; 0.088]
Deviance	36182.944	50030.207	9325.076	39948.002	138471.631
indep.obs.	8	11	17	8	44
participants	74	102	17	70	263
N	5328	7344	1224	5040	18936

Table 14: Mixed effects estimation of equation (21)

$\beta^{\text{opt}|\text{exp}} + \beta^{\text{opt}|\text{exp}}{}_{\text{X}}$=1.552 for the experienced subjects in the 'expectations with info' treatment. This means that the experienced bidders are slightly closer to the best replies given their expectations as compared to the inexperienced bidders.

3. **Experienced subjects overbid more in the 'no expectation' treatment:** Augmenting equation (4) with treatment-specific experience dummies that allow for shifts in the intercept reveals a significant effect of experience on overbidding in the baseline ('no expectations') treatment.

$$b_{ikt}(x) = \beta_0 + \beta^* \cdot b^*(x) + \sum_T \beta^T \cdot d^T + \sum_T \beta^T{}_{\text{X}} \cdot d_{\text{X}} d^T + \nu_i + \nu_k + \epsilon_{iktx} \qquad (22)$$

Table 15 presents estimation results.

	β	σ	t	p value	95% conf	interval
(Intercept)	-7.95	0.666	-11.9	0.0000	-9.26	-6.65
$b^*(x)$	1.7	0.00603	281	0.0000	1.68	1.71
expectations	3.15	1.1	2.87	0.0042	0.994	5.3
expectations w. info	2.69	0.958	2.81	0.0049	0.816	4.57
computer opp.	2.19	1.35	1.62	0.1053	-0.461	4.84
asymmetric	0.601	0.984	0.61	0.5415	-1.33	2.53
d_{X}no expectations	2.57	1.02	2.53	0.0113	0.582	4.56
d_{X}expectations	-1.31	1.03	-1.27	0.2030	-3.32	0.706
d_{X}expectations w. info	-0.699	0.943	-0.741	0.4588	-2.55	1.15
d_{X}computer opp.	-1.36	4.69	-0.29	0.7717	-10.5	7.83
d_{X}asymmetric	2.49	1.94	1.28	0.2001	-1.32	6.3

Table 15: Mixed effects estimation of equation 22

4. **Expectations of experienced subjects respond stronger to changed bids of other bidders:**

Equations (7) regresses the change of stated expectations on the change of the bid strategies played by other bidders. This allows to see if bidders pay attention to the bidding behaviour of their opponents when forming expectations. Equation (8) adds the change in own bids to the regressors. Here we augment both specifications by three experience dummies that allow for shifts in the intercept (d_X) and that allow for different slopes for the change in other bids (${\beta^\mathrm{other}}_\mathrm{X}$) and own bids (${\beta^\mathrm{own}}_\mathrm{X}$).

$$\begin{aligned}\Delta_t b^\mathrm{exp}_{ikx} &= \beta^\mathrm{other} \cdot \Delta_{t-1} b^\mathrm{other}_{ikx} + {\beta^\mathrm{other}}_\mathrm{X} d_\mathrm{X} \Delta_{t-1} b^\mathrm{other}_{ikx} + \\ &\quad + \beta_0 + \beta_{0\mathrm{X}} d_\mathrm{X} + \nu_i + \nu_k + \epsilon_{iktx} \qquad (23)\\ \Delta_t b^\mathrm{exp}_{ikx} &= \beta^\mathrm{other} \cdot \Delta_{t-1} b^\mathrm{other}_{ikx} + {\beta^\mathrm{other}}_\mathrm{X} d_\mathrm{X} \Delta_{t-1} b^\mathrm{other}_{ikx} + \\ &\quad + \beta^\mathrm{own} \cdot \Delta_t b^\mathrm{own}_{ikx} + {\beta^\mathrm{own}}_\mathrm{X} d_\mathrm{X} \Delta_t b^\mathrm{own}_{ikx} + \beta_0 + \nu_i + \nu_k + \epsilon_{iktx} \qquad (24)\end{aligned}$$

Table 16 provides estimation results showing that the intercept for the experienced bidders does not differ significantly from the one estimated for the inexperienced subjects. Further, changed expectations of the experienced bidders respond more pronouncedly to changes in the bids of other subjects in both specifications.

	(23)	(24)
(Intercept)	0.316**	0.290***
	[0.105; 0.526]	[0.147; 0.432]
β^other	0.043***	0.048***
	[0.036; 0.051]	[0.038; 0.058]
d_X	0.008	−0.020
	[−0.405; 0.421]	[−0.286; 0.247]
${\beta^\mathrm{other}}_\mathrm{X}$	0.026**	0.038**
	[0.007; 0.046]	[0.015; 0.061]
β^own		0.290***
		[0.271; 0.308]
${\beta^\mathrm{own}}_\mathrm{X}$		−0.057*
		[−0.108; −0.007]
Deviance	63233.503	58711.148
indep.obs.	19	19
participants	172	172
N	9950	9620

Table 16: Mixed effects estimation of equations (23) and (24)