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Local and Group Interaction in Prisoners' Dilemma Experiments*

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Abstract

This study investigates experimentally the effects of locality on learning and strategic behaviour in a repeated prisoners' dilemma. We compare a simple situation that models space (players interact only with their neighbours) with one that does not (players interact with all members of the population). Within this context different information conditions are studied. We find that imitation, while assumed to be a driving force in many models of spatial evolution, is a negligible factor in the experiment. Behaviour is driven by reinforcement learning and, under specific conditions, strategic behaviour.

JEL-Classification: C72, C92, D74, D83, H41, R12

Keywords: Local interaction, experiments, prisoners' dilemma, reinforcement, learning.

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1 Introduction

Space has been regarded as being crucial for many economic situations not only since Hotelling (1929). Restaurants or shops along streets do not compete equally with all other restaurants or shops on that street. Strategic interaction and imitation is more important among producers of similar products. Nevertheless, economists often have to simplify problems by neglecting geographic space or product space and only studying a situation where all agents influence each other in the same way. The latter is what we call a ‘non-spatial’ situation. However, over the last few decades there has been a growing awareness for the need to introduce space into economic reasoning in particular by evolutionary game theorists (see Axelrod (1984), Nowak and May (1992), Nowak and May (1993), Bonhoeffer, May, and Nowak (1993), Lindgreen and Nordahl (1994), Eshel, Samuelson, and Shaked (1998), Kirchkamp (1999), Kirchkamp (2000)). A large part of this literature concentrates on prisoners’ dilemmas and shows how imitating players may end up cooperating in spatial structures whereas they will not cooperate in a spaceless structure. In these imitation models space serves two purposes: Space reduces the possibility to exploit cooperating players (they can only be exploited by their neighbours) but space also reduces the visibility of successful non-cooperators (again, they can only be imitated by their neighbours). We will give an example for such a process in section 3.

As an application we should expect that e.g. cartels were found mainly in industries where product space or geographic space is relevant for the interaction among firms.

In this paper we will investigate this theory with the help of experiments. In contrast to the theoretical prediction we will find in our experiments less cooperation in the spatial structure and not more. A detailed analysis of our data shows that there are two reasons for this finding: Firstly, players in our experiments do not imitate but learn mainly through reinforcement. As a result the level of cooperation in the spatial structure drops. Secondly, there is, under some conditions, a strategic reason to cooperate, but only in the spaceless structure. As a consequence the level of cooperation in the spaceless structure raises.

First experiments to investigate the role of space have been done with coordination games by Keser, Erhart, and Berninghaus (1997, 1998). They find that players choose payoff dominant equilibria in a non-spatial structure but risk dominant equilibria in a spatial structure. The context of coordination games, however, does not allow to easily differentiate between myopic optimisation¹, imitation, or reinforcement learning². A player in a coordination game who chooses the same strategy as his or her neighbours might do this as a result of learning through imitation or as a result of myopic optimisation. In coordination games both motives often imply the same action. In the current paper, we choose the context of a prisoners’ dilemma where myopic optimisation, learning from own experience and learning through imitation often recommend different actions and, thus, can be disentangled more easily.

Learning through reinforcement and imitation are concepts often referred to in evolutionary game theory. Reinforcement learning (Börgers and Sarin 1997, Erev and Roth 1998) assumes that players are more likely to repeat successful rather than unsuccessful own choices. Since in

¹Myopic optimisation has been analysed in a spatial context by Ellison (1993) and Berninghaus and Schwalbe (1996).

²Learning through imitation or reinforcement learning has been analysed in a spatial context by Kirchkamp (1999).

a prisoners' dilemma non-cooperation is always more successful than cooperation, pure reinforcement learning over actions in the stage game always leads to non-cooperation, regardless of the spatial structure.

Imitation extends learning to observed players. Players are more likely to copy successful rather than unsuccessful neighbours. A typical learning rule, *copy best average*,³ assumes that players calculate average payoffs of all strategies they observe in their neighbourhood (including their own payoff) and choose the strategy with the highest average payoff. The player's own experience gets the same weight as the experience of any neighbour. In other words, *copy best average* presupposes only a moderate amount of reinforcement but a lot of imitation.

In the following we will always assume that learning applies to the choice of actions in the stage game and not to the choice of repeated game strategies. Thus, we avoid the problem of identification of repeated game strategies from the data. Given that participants in our experiments have to choose actions in every round this approach seems to be appropriate.

We will describe the experimental setup in section 2. In section 3, we briefly summarise a theoretical argument that is based on a simple imitation model and that suggests more cooperation in a spatial world than in a non-spatial world. In section 4 we come to our experimental results, which indicate the opposite: less cooperation in space than in a non-spatial setting. We will study stage game behaviour in section 4.1. Section 4.2 studies reinforcement and imitation and section 4.3 strategic behaviour and the motive to reciprocate. A second experimental setup, where players are more focused on learning, will be introduced in section 4.4. We will find that in this setup differences between the spatial and non-spatial structure become much smaller. A third setup, where we seed the population with computerised cooperators, will be discussed in section 4.5. Section 5 concludes.

2 The experimental setup

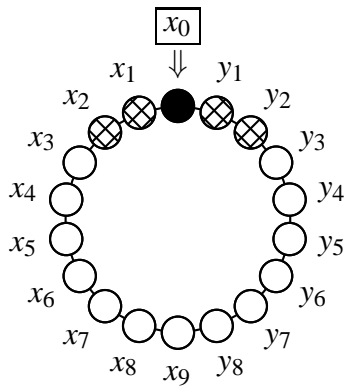
In the following, we describe results from 35 experiments in Barcelona and Mannheim, involving 339 participants and 5 different treatments. A list of these experiments is given in appendix A.

In the current section we will give a description of the first two treatments. One of them will be called a 'circle' treatment, the other 'group' treatment. The remaining three treatments are described in section 4.4 below.

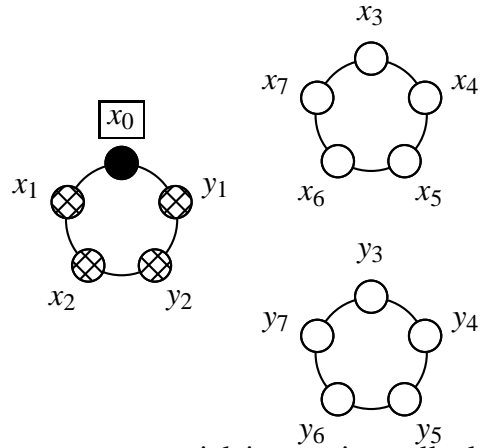
In the circle treatment we study a spatial structure of 18 players. Participants are randomly seated in front of computer terminals that are networked to create a neighbourhood structure (see left part of figure 1). Each player interacts in each round with two neighbours to the left and two neighbours to the right. Player x_0 , e.g., is in interaction with x_1, x_2 , and y_1, y_2 . Player x_2 , however, is in interaction with x_3, x_4 , and x_1, x_0 . Players are able to observe payoffs and strategies of each of their neighbours.

In the group treatment we study groups consisting of five players each. Each member of a group interacts in every round of the experiment with all members of the group (see right part of figure 1).

³See Axelrod (1984), Bonhoeffer, May, and Nowak (1993), Eshel, Samuelson, and Shaked (1998), Kirchkamp (2000), Lindgreen and Nordahl (1994), Nowak and May (1992), Nowak and May (1993).



Circle: spatial interaction of players through overlapping neighbourhoods



Groups: non-spatial interaction, all players are either in the same neighbourhood, or do not interact at all.

FIGURE 1: Neighbourhoods

Thus, both in the group and in the circle treatment the number of interaction partners is four. Theory⁴ predicts no cooperation in groups, independent of the size of the groups, and sometimes⁵ cooperation in circles, independent of the size of the neighbourhoods in circles. We are, hence, free to choose any group size for our comparison. For simplicity, we use the same size of neighbourhoods in both structures. Later, when we find that our experiment does not confirm the theoretical prediction, we can relate this deviation to strategic behaviour, imitation and reinforcement learning. Most importantly, imitation is much weaker than assumed in the theory and not strong enough to support cooperation. We have no reason to assume that this finding depends on the precise size of the neighbourhood.

During any session of the experiment, players always interact with the same neighbours. Sessions last for 80 periods. In each period participants play a prisoners' dilemma against all members of their neighbourhood/group. They can only use the same strategy against all neighbours/group members. The payoff table (table 1)⁶ is displayed on the participants' screen throughout the experiment. Playing *D* gives always a payoff that is 4 points higher than the payoff from playing *C*, but each *C* contributes 5 points to the payoff of each of his or her neighbours.

During the course of play players observe their own payoff and the payoff and actions of their neighbours. This takes place in circles and groups in the same way. In the following we give an example of the situation in a circle. Let us assume that a cluster of five players who choose action *C* is located within a larger cluster of players who all choose *D* as shown in table 2. Consider player 1 from table 2 who has two neighbours with action *C* and two other neighbours with *D*. The representation of payoffs in this period for this player is shown in

⁴E.g. imitating agents that use *copy best average* or *copy best* as a learning rule (Nowak and May 1992, Eshel, Samuelson, and Shaked 1998).

⁵Depending on payoffs and initial configuration.

⁶In the experiment *C* and *D* are represented in different colours (blue and red) and are actually called (neutrally) 'A' and 'B'. In some sessions *C* is red ('A') and *D* is blue ('B'), in others *D* is red ('A') and *C* is blue ('B').

Payoff:					
own action	number of	neighbours group members			choosing C
		0	1	2	
C	0	5	10	15	20
D	4	9	14	19	24

TABLE 1: Payoff Matrix

Player	1	2	...								
Neighbourhood of Player 1	-	-	↓	-	-								
Action:	...	D	C	C	C	C	C	D	D	D	D	D	...
# of other Cs in the neighbourhood	...	2	2	3	4	3	2	2	1	0	0	0	
Own payoff	...	14	10	15	20	15	10	14	9	4	4	4	
Average payoff of C		12.5	15	15	14	15	15	12.5	10	—	—	—	
Average payoff of D in the neighbourhood		9	11.5	14	—	14	11.5	9	7.75	7	5.25	4	

TABLE 2: Example of a neighbourhood of C s and D s

table 3. The player’s own payoff is 10 , which is displayed next to the player’s own action C . The player has two neighbours with action C and payoffs 20 and 15 respectively. The two other neighbours chose action D and receive payoffs 14 and 9 . Payoffs obtained with either C or D are displayed in different colours in the experiment. The payoffs are shown in the rightmost column and ordered from highest to lowest. Thus, it is not obvious to the player *which* of the player’s neighbours has chosen a certain action and received a certain payoff.

3 A simple imitation model

We will now sketch a simple and common evolutionary learning dynamics that is based on imitation⁷ and that suggests more cooperation in a spatial environment and less in a non-spatial one. From the example in this section, it should become clear that with imitation we should expect more cooperation in the spatial structure than in the non-spatial one. In section 4.1, however, we will experimentally find less cooperation in the spatial structure. We conclude that there must be another effect and introduce strategic behaviour as such an effect in section 4.3. We will further show in sections 4.2, 4.4, and 4.5 that players’ actions are not driven by imitation. With this argument we do not want to contradict evolutionary game theory. What we want to point out is that imitation, while theoretically being an interesting concept, does not seem to play an important role in our experiments.

⁷Similar dynamics are used e.g. in Bonhoeffer, Nowak, and May (1993), Eshel, Samuelson, and Shaked (1998), Kirchkamp (2000), Lindgren and Nordahl (1994), Nowak and May (1992, 1993).

History						
R	Your		your neighbours			
O	strategy		received			
U	and					
N	gains					
D	are					
...
...	C	10	20	15	14	9

Payoffs of Cs are shown in a box, payoffs of Ds are shown in gray. In the experiment we use the colours red and blue (randomly assigned to C and D).

TABLE 3: Example of payoff representation for player 1 from table 2.

...
...	D	14	15	10	9	4

Payoffs of Cs are shown in a box, payoffs of Ds are shown in gray. In the experiment we use the colours red and blue (randomly assigned to C and D).

TABLE 4: Example of payoff representation for player 2 from table 2.

Obviously, in a non-spatial setting with myopic imitation, or replicator dynamics, non-cooperation is always more successful than cooperation. Hence, in a non-spatial setting, cooperation always dies out. In the upper part of figure 2 we give an example. We simulate a group of five players who always imitate the strategy with the highest average payoff in their neighbourhood (*copy best average*). With a small probability (1% in this example) players ‘mutate’ and chose the other strategy. We start with 5 cooperating players who imitate cooperation until in period 13 one player mutates and plays D. Being very successful, this player is imitated by all neighbours and from period 14 on everybody plays D. Further mutants that appear in later periods do not lead the group back to cooperation.

With spatial learning (Axelrod 1984, Bonhoeffer, May, and Nowak 1993, Eshel, Samuelson, and Shaked 1998, Kirchkamp 2000, Lindgreen and Nordahl 1994, Nowak and May 1992, Nowak and May 1993) however, cooperation is protected through space and may, hence, survive.⁸ Consider player 2 from table 2. The representation of payoffs for this player in the experiment is shown in table 4. An imitating D with this information finds C to obtain higher payoffs (15 and 10) than D (9 and 4) and may decide to become a C — thus, cooperation may survive or grow⁹. Also in our example (see the bottom part of figure 2) cooperation grows from the initial configuration of only five Cs and is not much affected by mutants.

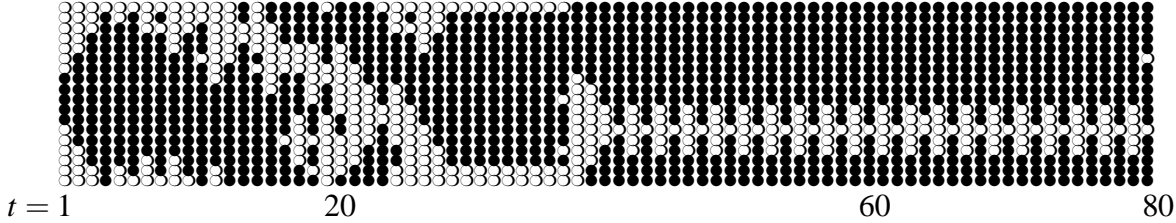
⁸With myopic optimisation (Ellison 1993) players would obviously never cooperate.

⁹Once the cluster of Ds becomes small the payoff of the remaining Ds grows and the process stops or enters a cycle. With standard imitation processes stable equilibria are often reached when clusters of successful Cs are separated by small clusters of equally successful Ds.

‘Copy best average’ imitation in a group:



‘Copy best average’ imitation in a circle:



● = C, ○ = D. The first mutant D makes cooperation disappear completely in groups. Cooperation in circles, however, persists despite mutant Ds.
 (The imitation rule is ‘copy best average payoff’, the mutation rate is 1%, the imitation and interaction radius is 2, as in the experiment. Simulations starts with 5 cooperators in the first period.)

FIGURE 2: Simulated learning.

4 Results

In a first treatment we ran 5 sessions, each lasting for 80 rounds. In this treatment players are located in a circle, know the payoff matrix (table 1) and receive per individual payoff feedback as in table 3. The players’ actions are shown in appendix B.1. In a second treatment we ran 10 sessions, with players now located in groups of 5 players each. The information given was as in the first treatment. Actions are displayed in appendix B.2.

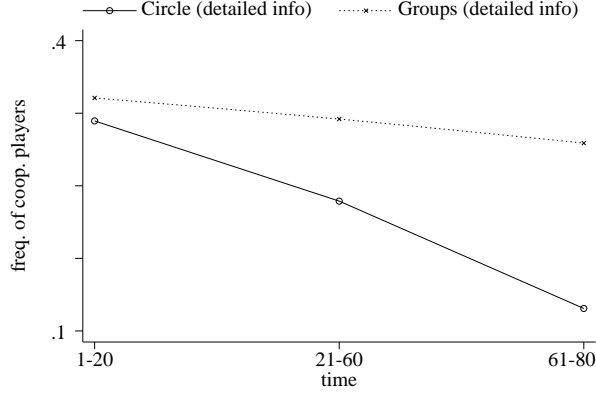
In section 4.1 we study stage game behaviour and find that in contrast to the simple imitation dynamics discussed in section 3 there is more cooperation without space (in groups) than with space (in circles). In section 4.2 we study learning, which in both structures seems mainly be driven by reinforcement and not by imitation. Section 4.3 introduces strategic behaviour and gives an explanation for the larger amount of cooperation in groups. In sections 4.4 and 4.5 study a modified treatment where we eliminated strategic behaviour. Again we find that players learn mostly through reinforcement and not through imitation. As a result cooperation is not supported through the spatial structure.

4.1 Cooperative behaviour

In figure 3 we show the relative frequency of cooperation in circles and groups. At the beginning of the treatments levels of cooperation are about the same in groups and in circles and similar to what is found in other non-spatial experiments¹⁰.

¹⁰Bonacich et. al. (1976) studied cooperation within groups of 3, 6, and 9 players in a game where cooperation is less attractive than in our game. They found levels of about 30% of cooperation in groups, which is close to the results in our experiment.

Fox and Guyer (1977) used a non-linear payoff scheme where sometimes cooperation was more attractive than in our game. They found more cooperation (around 50%) in a game with groups of 3 and 12 players.



The figure shows the average frequency of cooperation for the first 20 periods, period 20–60, and for period 61–80.

FIGURE 3: Frequency of cooperative players in circles and groups over time

During the treatment the frequency of cooperating players decreases substantially in circles and remains more or less constant in groups. We find that the frequency of cooperation in the last 60 periods of the experiment is significantly lower in circles than in groups¹¹. This finding contradicts the prediction of the imitation model that we sketched in section 3. There we motivated more cooperation in circles due to learning from neighbours. Since here we find less cooperation we will first check in section 4.2 whether players’ learning behaviour fits the properties assumed in section 3. We will find an explanation for the low levels of cooperation in circles, but we will still miss an explanation for the relatively high levels of cooperation in the spaceless structure. To fill this gap we will then in section 4.3 study reciprocity.

4.2 A simple model of reinforcement and imitation

We estimate the following logit model:

$$P(c_{t+1}) = \mathcal{L}(\beta_0 + \beta^{\text{own}} u_t^{c,\text{own}} + \beta^{\text{other}} u_t^{c,\text{other}}) \quad (1)$$

where $\mathcal{L}(x) = e^x / (1 + e^x)$, c_{t+1} is 1 if a player cooperates tomorrow, and 0 otherwise, $u_t^{c,\text{own}}$ is the player’s own payoff from cooperation as seen in period t and $u_t^{c,\text{other}}$ is the player’s neighbour’s payoff from cooperation as seen in period t ¹². The factor β_{own} captures, hence, reinforcement, β_{other} measures the amount of imitation, and β_0 a general inclination to cooperate.

¹¹A t-test yields $t = 3.20$, $P_{>|t|} = 0.006$. When calculating levels of standard deviations and levels of significance we have to take into account that observations within any of our experiments may be correlated. We can, however, assume that covariances of observations from different experiments are zero. Covariances of observations from the same experiment are replaced by the appropriate product of the residuals (Rogers 1993). We will use this approach throughout the paper to calculate standard errors.

Still, we can also calculate a two-sample Wilcoxon rank-sum (Mann-Whitney) test and find a $z = 2.327$, $P_{>|z|} = 0.0200$.

¹²If a player does not cooperate in a given period t the value of $u_t^{c,\text{own}}$ can not directly be determined. In this case we recursively use $u_t^{c,\text{own}} := u_{t-1}^{c,\text{own}}$ until we reach a period where the player actually cooperated. In the same

coeff. from eq. (1)	Circles					
	β	σ	t	$P_{> t }$	95% conf. interval	
$u^{c,own}$.0962337	.0066259	14.52	0.000	.0832471	.1092203
$u^{c,other}$.0682104	.005713	11.94	0.000	.057013	.0794077
c	-1.951959	.0357078	-54.66	0.000	-2.021945	-1.881973
	Groups					
$u^{c,own}$.0509183	.0082287	6.19	0.000	.0347903	.0670463
$u^{c,other}$.0794446	.0078832	10.08	0.000	.0639938	.0948954
c	-1.58941	.0558171	-28.48	0.000	-1.69881	-1.480011

TABLE 5: GEE population-averaged estimation of equation (1)

coeff. from eq. (1)	Circles					
	β	σ	t	$P_{> t }$	95% conf. interval	
$u^{c,own}$.088943	.0056999	15.60	0.000	.0777713	.1001147
$u^{c,other}$.1040525	.0052749	19.73	0.000	.0937138	.1143912
c	-2.062085	.0304381	-67.75	0.000	-2.121743	-2.002428
	Groups					
$u^{c,own}$.0140842	.0067888	2.07	0.038	.0007785	.0273899
$u^{c,other}$.1626808	.007345	22.15	0.000	.1482849	.1770767
c	-1.743681	.0427707	-40.77	0.000	-1.82751	-1.659852

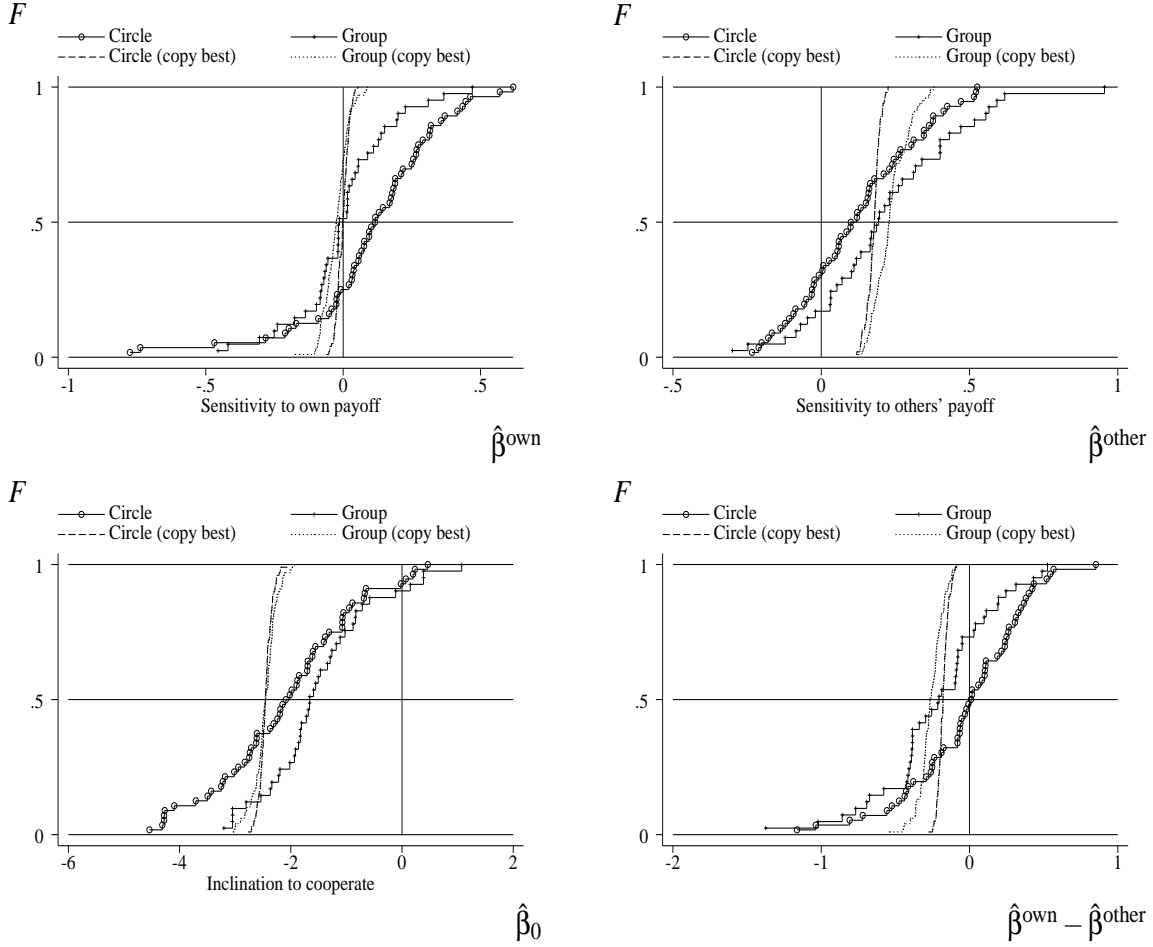
TABLE 6: Logit estimation of equation (1)

We do three estimations of the above model. One is a GEE population-averaged model (Liang and Zeger 1986) that takes into account the AR(1) nature of the process¹³. We use as a link function the logistic function and specify c_{t+1} to be binomially distributed. Results are shown in table 5. As a reference we also show a standard logit estimation that does not take into account the AR(1) process in table 6. We see that qualitatively the relations between the coefficients remain the same within the two estimations. This justifies that we can take the standard logistic model for the individual estimates where the GEE population-averaged model can not be applied. The cumulative distribution for the coefficients that were, in a third approach, estimated separately for each individual, is shown in figure 4. As a reference for what evolutionary game theory would assume (see section 3) the figure includes estimates for 100 simulated populations where players follow the copy-best-average rule.

As we see in figure 4 players are more sensitive to their own payoff in experiments than the simulated players who use a copy-best-average rule. Likewise they are less sensitive to their neighbours' payoff. Table 7 summarises tests for equality of β^{own} , respectively β^{other} , in the

way we define recursively $u_t^{c,other} := u_{t-1}^{c,other}$ until we reach a period where the player actually cooperated to define $u_t^{c,other}$. For the estimation we have dropped the first 20 periods to give participants some time to learn.

¹³We use the AR(1) approach since we should expect correlations within the variables of our approach. The dependent variable c_{t+1} influences the explanatory variable $u_t^{c,other}$ in the next period. Since the size of the impact depends on the number of other cooperators and varies from period obtaining a perfect estimation is hardly possible but also not necessary since we are only interested in relative magnitudes of the coefficients.



Solid lines show the cumulative distribution of individual estimates for each participant in our experiment. Dotted and dashed lines show results for 100 simulated populations following the learning rule copy-best-average with a mutation rate of 0.1.

FIGURE 4: Cumulative distribution of estimated coefficients from equation (1).

	$\beta_{\text{Copy best}}^{\text{own}} = \beta_{\text{Experiment}}^{\text{own}}$	$\beta_{\text{Copy best}}^{\text{other}} = \beta_{\text{Experiment}}^{\text{other}}$
Circles	$t = -4.48 \quad P_{<t} = 0.000$	$t = 4.55 \quad P_{>t} = 0.000$
Groups	$t = -1.84 \quad P_{<t} = 0.034$	$t = 1.72 \quad P_{>t} = 0.043$

Tests are based on the coefficients that were estimated for each individual separately. Outliers were eliminated using Hadi's method (Hadi 1992, Hadi 1994). When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11).

TABLE 7: Learning in the experiment versus copy best average

coeff. from eq. (1)	robust regression						rank sum test	
	β_c	σ_b	t	$P_{> t }$	95% conf. interval		z	$P_{> z }$
$\hat{\beta}^{\text{own}}$.1063097	.0486674	2.18	0.046	.0019286	.2106908	2.082	0.0373
$\hat{\beta}^{\text{other}}$	-.0947037	.0402311	-2.35	0.034	-.1809909	-.0084165	-1.837	0.0662
$\hat{\beta}_0$	-.5807385	.2316169	-2.51	0.025	-1.077507	-.0839696	-2.082	0.0373
$\hat{\beta}^{\text{own}} - \hat{\beta}^{\text{other}}$.2010135	.0618886	3.25	0.006	.0682756	.3337513	2.327	0.0200

Outliers were eliminated using Hadi's method (Hadi 1992, Hadi 1994). When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11).

Estimated coefficients show differences between circles and groups.

TABLE 8: Estimation of equation (2) for coefficients from equation (1)

experiment and in the simulations. In particular for circles the results are highly significant.

Remember that in section 3 we explained that survival of cooperation crucially depends on learning from neighbours. Finding only a small amount of cooperation in circles should, hence, not come at a surprise, given that player do not imitate their neighbours.

To study differences between groups and circles we estimate for each coefficient $\beta^{\text{own}}, \beta^{\text{other}}, \beta_0$ the following robust regression

$$b = c + \beta_c d_c \quad (2)$$

where b is the estimation of one the coefficients from equation (1), c is a constant, d_c is a dummy that is one for circles and zero otherwise. Results are shown in table 8.

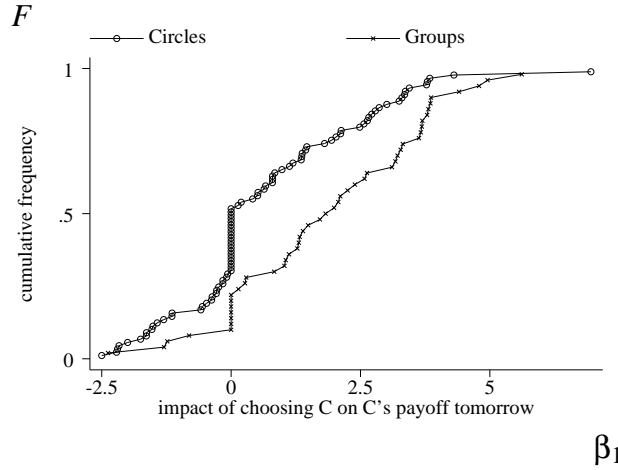
We see that learning behaviour in circles is significantly different from learning in groups. In particular the coefficient $\hat{\beta}^{\text{other}}$ is significantly smaller in circles than in groups, i.e. participants in circles are significantly less sensitive to their neighbours' payoff than participants in groups. This finding is in line with what we should expect theoretically for learning rules in a spatial environment. A neighbour's success with a strategy may be due to this neighbour's neighbourhood and might not apply to the learning players (Kirchkamp 2000). Hence, a player living in a spatial structure should be less sensitive to his or her neighbour's payoffs than one in a non-spatial one. Consistent with this finding also the difference $\hat{\beta}^{\text{own}} - \hat{\beta}^{\text{other}}$ is significantly larger in circles.

In the current section we have found an explanation for the small amount of cooperation in circles. What we are still missing is an explanation for the relatively large amount of cooperation in groups. We will give this explanation in section 4.3.

4.3 Reasons to speculate for reciprocity

What reasons does a player have to choose C ? In this section we show that choosing C increases payoff in groups but not in circles. To do that we estimate

$$u_{t+1}^{c,\text{all}} = \beta_0 + \beta_1 c_t + \beta_2 c_{t+1} \quad (3)$$



The figure shows the cumulative frequency of the individual estimates of the coefficient β_1 from equation (3), separately for groups and circles.

FIGURE 5: Impact of choosing C , depending on the structure

	robust regression					rank sum test		
	β	σ_b	t	$P_{> t }$	95% conf. interval	z	$P_{> z }$	
β_g	1.311531	.2780627	4.72	0.000	.7151455	1.907916	2.449	0.0143
c	.7638131	.0951821	8.02	0.000	.5596678	.9679585		

Outliers were eliminated using Hadi's method (Hadi 1992, Hadi 1994). When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11).

TABLE 9: Result of estimating equation (4)

where $u_{t+1}^{c,all}$ is the average payoff of all C s in the neighbourhood in period $t + 1$, c_t is one if a player cooperates in period t and zero otherwise and c_{t+1} is one if a player cooperates in period $t + 1$ and zero otherwise¹⁴.

The cumulative frequency of the individual estimates of the coefficient β_1 is shown in figure 5. We see that the distribution of the β_1 s is located more to the right in groups. To test this more formally we estimate the following model

$$\hat{\beta}_1 = c + \beta_g d_g \quad (4)$$

where $\hat{\beta}_1$ is the value of β_1 from equation (3) that we estimated for each participant separately, c is a constant, and d_g is a dummy that is one for groups and zero otherwise. The results are shown in table 9. The coefficient β_g measures the additional gain from C in groups. We find that players in groups have significantly more reasons to choose C than players in circles. ($P_{>|t|} = 0.0003$)¹⁵. Also a two-sample Wilcoxon rank-sum (Mann-Whitney) test finds the difference

¹⁴Using the average payoff from C as a dependent variable and not the individual payoff has the advantage that our result is less influenced by what the player does at $t + 1$.

¹⁵When calculating levels of standard deviations and levels of significance we take into account that observations

History			
R	Your	in your	
O	strategy	neighbourhood	
U	and	the average payoff	
N	gains	was with...	
D	are	C	D
...
...	D 14	12.5	9
...

TABLE 10: Representation of payoffs in the ‘less-information’ treatment

highly significant when comparing the 10 average values for β_1 in groups with the 5 average values for β_1 in circles ($z = 2.449$, $P_{>|z|} = 0.0143$).

Here we only measure how players can influence the reciprocal behaviour of their neighbours and what this means for their own behaviour. A more detailed discussion of why this could be the case can be found in Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) who relate reciprocity to utility from own and others’ payoff.

4.4 An experiment with less information

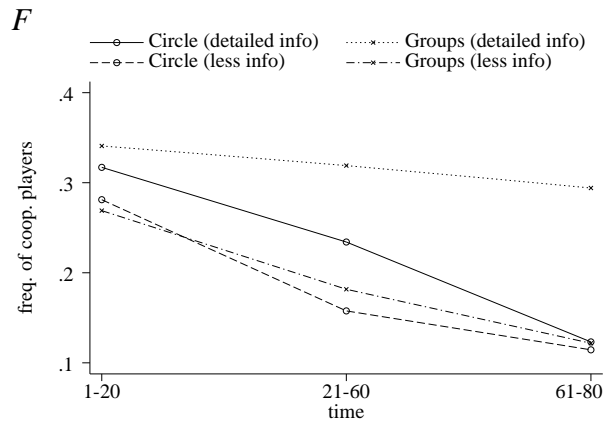
In section 4.3 above we have found that participants in the groups-treatment of our experiment reciprocate significantly stronger than those in the circle-treatment. Why is this the case? Reciprocity requires an understanding of the strategic situation, in particular of the prisoners’ dilemma nature of the game. To establish a setting where reciprocity does not play a role, we decided to run a series of experiments with less information about the strategic situation but similar possibilities to learn. Instead of presenting the information as described in section 2 we never told subjects the payoff matrix (table 1) in this treatment. Further we did not give the per individual payoff feedback as in table 3 that would allow to quickly reconstruct the payoff matrix. Subjects only knew that they were playing a symmetric game and received information about average payoffs of the two strategies in their neighbourhood in each round as shown in table 10. We will call this condition in the following ‘less information’ in contrast to the ‘detailed information’ discussed in the previous sections.

Appendices B.3 and B.4 show the raw data for this condition in the circle and groups treatment respectively. In figure 6 we show how the relative frequency of cooperation develops in circles and groups over time. We see that once players are more focused on learning and less on strategic considerations, levels of cooperation are no longer significantly different¹⁶.

We estimate again equation (3), now for the less information treatment. The distribution of $\hat{\beta}_1$ is shown in figure 7. We see that the modification of our setup successfully eliminated most of the potential for strategic behaviour. Playing C no longer affects significantly payoffs

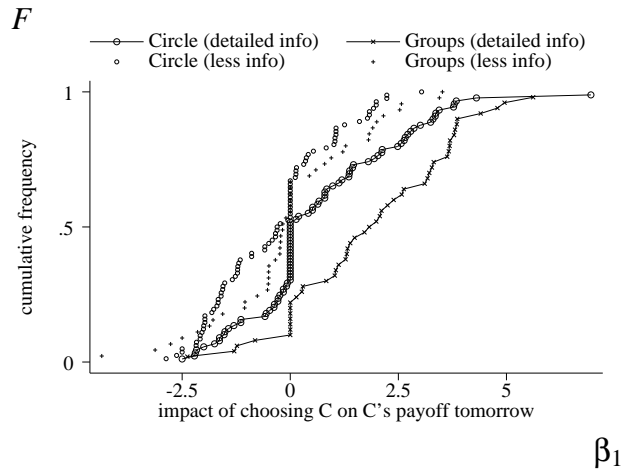
within any of our experiments may be correlated (see footnote 11).

¹⁶When testing whether the relative frequency of cooperation in the less-information treatment differs in circles from the one in groups we do not find a significant difference: $t = -0.79$, $P_{>|t|} = 0.445$. A two-sample Wilcoxon rank-sum (Mann-Whitney) comes to a similar conclusion: $z = 0.467$, $P_{>|z|} = 0.6404$.



The figure shows the average frequency of cooperation in circles and groups for the first 20 periods, period 20–60, and for period 61–80.

FIGURE 6: Frequency of cooperative players over time in the full information and in the less information treatment



The figure shows the cumulative frequency of the individual estimates of the coefficient $\hat{\beta}_1$ from equation (3), separately for groups and circles and separately for the two information conditions.

FIGURE 7: Impact of choosing C, depending on information and structure

	robust regression					rank sum test		
	β	σ_b	t	$P_{> t }$	95% conf. interval		z	$P_{> z }$
$\hat{\beta}_g$.3640991	.3496799	1.04	0.317	-.3913383	1.119537	0.467	0.6404
\hat{c}	-.3679798	.2642448	-1.39	0.187	-.9388459	.2028864		

TABLE 11: Result of estimating equation (4) in the less information treatment

coeff. from eq. (1)	robust regression					rank sum test		
	$\hat{\beta}_c$	σ_b	t	$P_{> t }$	95% conf. interval		z	$P_{> z }$
$\hat{\beta}^{\text{own}}$.0840038	.0686664	1.22	0.247	-.06713	.2351376	-0.081	0.9352
$\hat{\beta}^{\text{other}}$.0818402	.0517858	1.58	0.142	-.0321396	.19582	0.569	0.5691
$\hat{\beta}_0$	-.0021636	.043275	-0.05	0.961	-.0974112	.093084	0.407	0.6842
$\hat{\beta}^{\text{own}} - \hat{\beta}^{\text{other}}$	-.6018681	.2399373	-2.51	0.029	-1.129967	-.0737696	-2.034	0.0420

When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11).

TABLE 12: Estimation of equation (2) for coefficients from equation (1) in the less information treatment

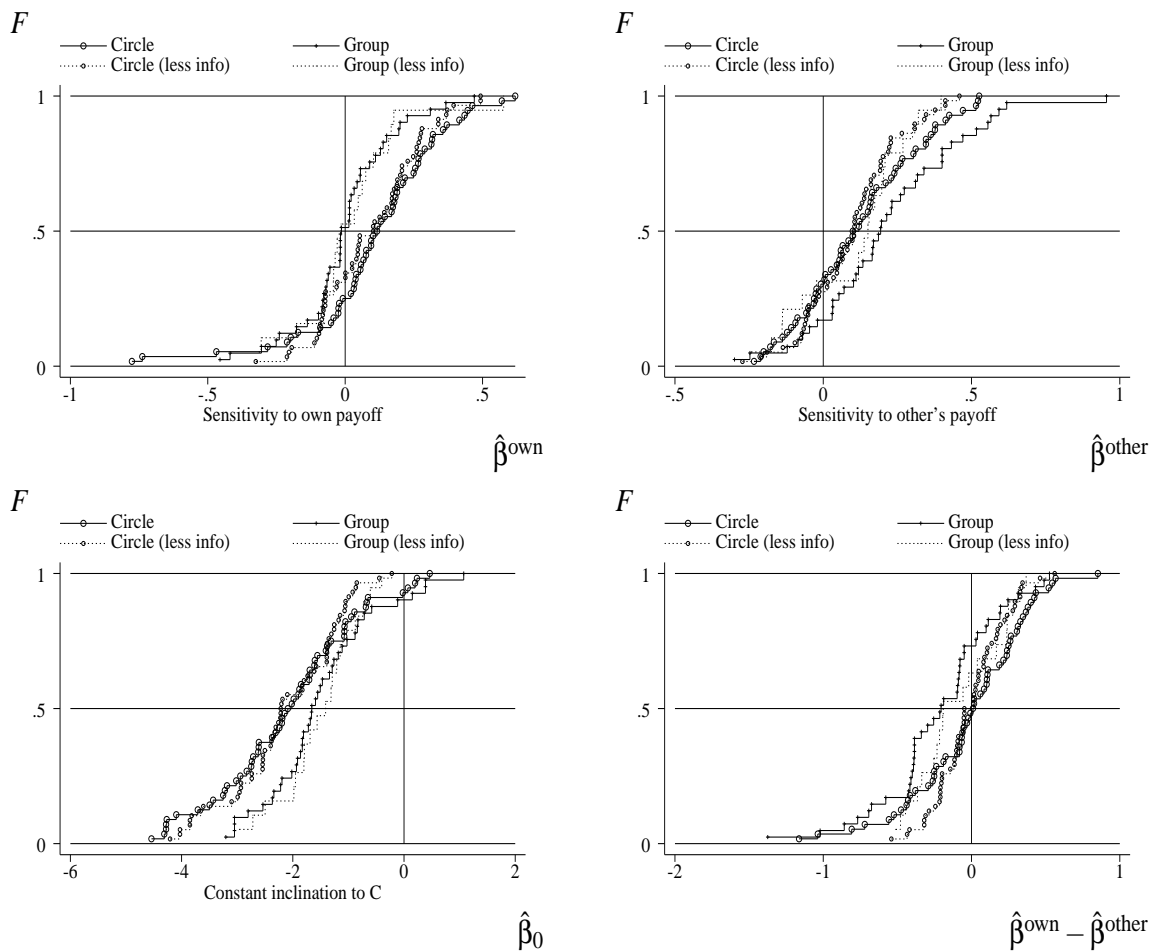
differently in groups or in circles. To make this argument more formally we estimate equation (4) also for the less-information condition and show the results in table 11. The coefficient $\hat{\beta}_g$ that measures the additional gain from C in groups is positive under the less-information condition but no longer significantly different from zero¹⁷. Also a two-sample Wilcoxon rank-sum (Mann-Whitney) test finds the difference not significant ($z = 0.467$, $P_{>|z|} = 0.6404$).

To study reciprocity and imitation in the less-information condition we estimate again equation (1) and show the results in figure 8. We can use the approach from equation (2) to test for significant differences among the estimated coefficients in the different treatments. Results are shown in table 12. Estimated coefficients are no longer significantly different in the two treatments, only the difference $\hat{\beta}^{\text{own}} - \hat{\beta}^{\text{other}}$. Apparently participants still learn more from their neighbours in groups than in circles.

4.5 Cooperation in seeded circles

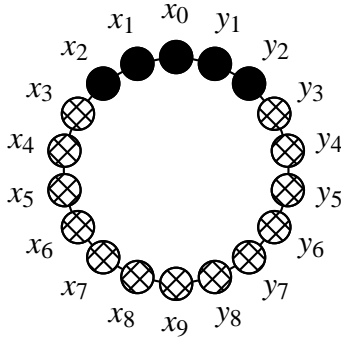
In section 3 we explained how imitation of successful neighbours supports cooperation in a spatial environment. This argument relies on the assumption of an initial cluster of cooperators of sufficient size — with our payoffs we need at least five neighbouring cooperators. But how does such a cluster appear? An evolutionary game theorist might argue that we only have to wait long enough until such a cluster appears with a mutation. Experiments, however, last only for a limited number of periods, and if the cooperative cluster does not appear during this time cooperation might never start. We therefore seeded a circle with a cluster of five computerised players as shown in figure 9. Players x_2, x_1, x_0, y_1, y_2 (the ‘seeds’) are played by the computer

¹⁷When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11).



Solid lines show the cumulative distribution for the treatment with detailed payoff information. Dotted lines show the distribution for the treatment with less payoff information.

FIGURE 8: Cumulative distribution of estimated parameters from equation (1) with detailed and with less payoff information.



The five black dots indicate the position of computerised players that always play C. The remaining dots indicate the position of the human players.

FIGURE 9: The structure of seeded circles

and cooperate in every period.¹⁸ The remaining players are human which obtain the same information as in the less-information treatment (section 4.4). Players x_3, x_4, y_3, y_4 do not know that their neighbours are computers. The details of the behaviour of the human players is shown in appendix B.5. Figure 10 shows the development of cooperation in this treatment. The dotted line shows, as a reference, the relative frequency of cooperative players in groups. The other lines show the development in circles. For the seeded circle we show two lines. The upper one shows all participants, including those that have immediate neighbours in the seeding cluster. These participants cooperate more than the remaining participants. When we exclude them, we obtain the lower line.

As long as we exclude those players that are immediate neighbours of the seeds we do not find a significant difference to the unseeded circles¹⁹. Those players who are immediate neighbours of the seeds, however, cooperate significantly more frequently²⁰.

The impact of strategic behaviour is under this condition similar to the other experiments with less information. We estimate for all participants in the less information circle treatment

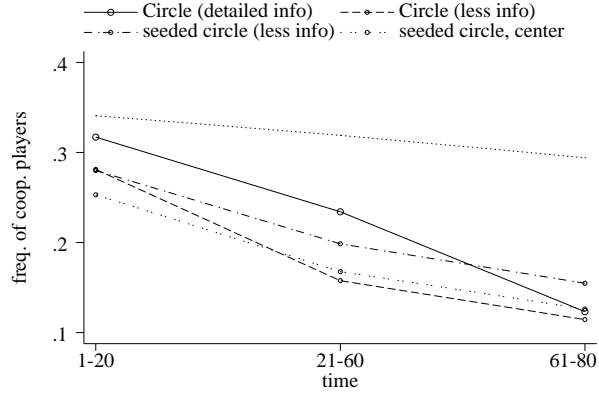
$$\hat{\beta}_1 = c + \beta_s d_s \tag{5}$$

where $\hat{\beta}_1$ is the value of β_1 from equation (3) that we estimated for each participant separately, c is a constant, and d_s is a dummy that is one for seeded circles and zero otherwise. The results are shown in table 13. We find that the seeding even reduces, however not significantly, the increase in payoff that results from cooperation.

¹⁸Participants were told that would play a game with 18 players sitting round a circle. They could see that only 13 player were present in the laboratory. We were prepared not to answer any question regarding the five missing players but in our experiments no participant ever missed them.

¹⁹ $t = 0.45, P_{>|t|} = 0.663$, When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11). A rank sum test finds $z = -0.915, P_{>|z|} = 0.3602$

²⁰ $t = 2.45, P_{>|t|} = 0.058$. When calculating levels of standard deviations and levels of significance we take into account that observations within any of our experiments may be correlated (see footnote 11). A rank sum test finds only $z = 1.441, P_{>|z|} = 0.1495$.



The line “seeded circle (less info)” shows the relative frequency of all human players, the line “seeded circle, center” excludes players x_3, x_4, y_3, y_4 .

FIGURE 10: Cooperation in seeded circles

	robust regression					rank sum test		
	β	σ_b	t	$P_{> t }$	95% conf. interval		z	$P_{> z }$
β_s	-.1360373	.3340148	-0.41	0.692	-.8802685	.608194	0.549	0.5830
c	-.3679798	.2669898	-1.38	0.198	-.9628701	.2269106		

TABLE 13: Result of estimating equation (5) in a seeded circle

5 Summary

We have studied the impact of reinforcement, learning and strategic behaviour in two structures, a spatial and a non-spatial one. Our initial aim was to find support for the imitation hypothesis that is assumed in a large part of the theoretical literature on the evolution of local interaction. This literature explains survival of cooperation in space based on the assumption that players imitate successful neighbours. Since a cluster of cooperators is more successful than a cluster of non-cooperative players imitation would yield a large part of the population to behave cooperatively.

Players do, indeed, learn in our experiment. However, they mostly learn through reinforcement, i.e. from their own payoff, and not through imitation of neighbours. If players do only learn from their own experience cooperation does not spread through a local structure. This finding is in line with the small amount of cooperation that we find in our experiments within a local structure.

The lack of learning from others, however, does not explain the comparatively large amount of cooperation in a spaceless structure — much more than in the local spatial structure. We found that this large amount of cooperation in the spaceless structure can be related to strategic behaviour. So far we do not know why strategic behaviour plays a more important role in the spaceless structure than in the spatial one. One explanation is that the strategic impact of the spatial structure is harder to understand than the spaceless one and, hence, players rely more on learning.

We then tried to focus participants in both structures in the experiment more on learning. In a new series of experiments we withheld the payoff matrix and also the history of individual payoffs in the neighbourhood. Participants could only see average payoffs of the two strategies. This led to a significant change for the spaceless structure where cooperation dropped down to the level in the spatial structure. Behaviour in the spatial structure, however, was seemingly unaffected. This confirms that behaviour in the spatial structure was driven by learning all the time, only in the spaceless structure strategic behaviour played a role as long as participants were not focused on learning.

As a third major modification of our setup we analysed a seeded population where a cluster of computerised players cooperated all the time. If players would imitate successful neighbours then such a cluster should be enough to initiate the growth of cooperation all over the network. In our experiment, indeed, the immediate neighbours of this cluster were only slightly but significantly more inclined to cooperate. This effect was so small that it could not reach farther into the network.

To summarise: We have studied three effects that are commonly used to explain behaviour in spatial and spaceless games: Reinforcement, imitation, and strategic behaviour. We found reinforcement to be a strong effect that was present in all conditions. Imitation was a much weaker effect, in particular in the spatial structure, and too weak to support cooperation. Strategic behaviour, only present in the spaceless structure, could easily be eliminated through a change in the available information.

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A List of Experiments

OVERVIEW:

Number of sessions in different treatments			
	information . . .		5 computerised cooperators
	detailed	less	
group	9	10	
circle	5	5	6

PARAMETERS OF EACH SESSION:

	structure	information	computerised cooperators	number of players
1.	Group	less info	0	5
2.	Group	less info	0	5
3.	Group	less info	0	5
4.	Group	less info	0	5
5.	Group	less info	0	5
6.	Group	less info	0	5
7.	Group	less info	0	5
8.	Group	less info	0	5
9.	Group	less info	0	5
10.	Group	detailed info	0	5
11.	Group	detailed info	0	5
12.	Group	detailed info	0	5

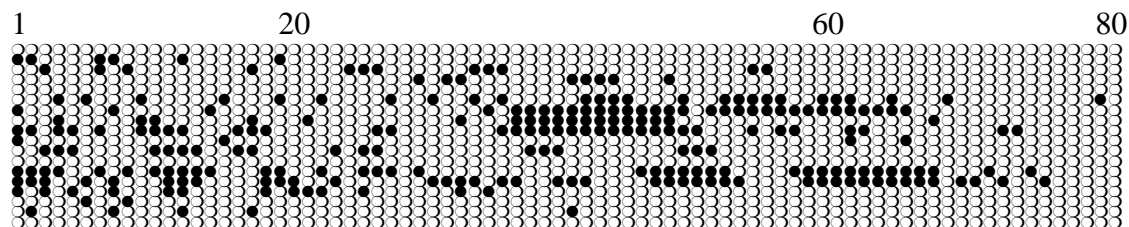
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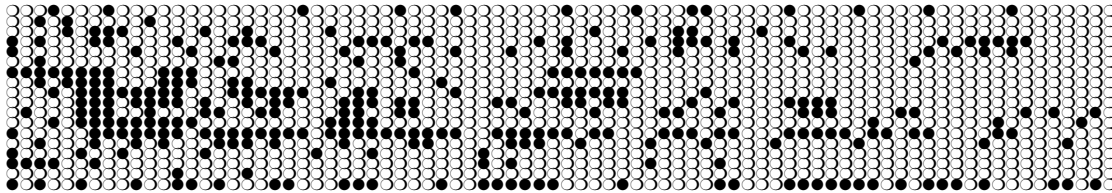
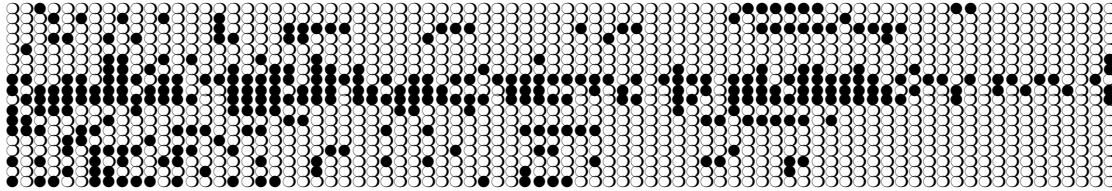
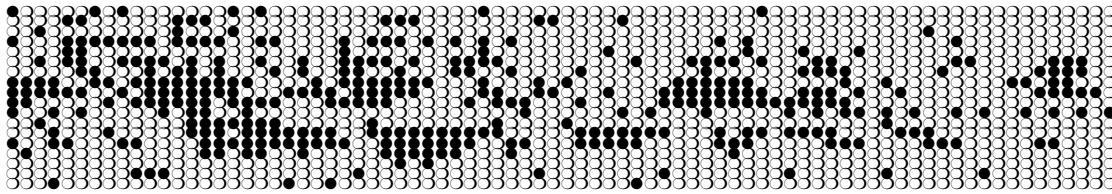
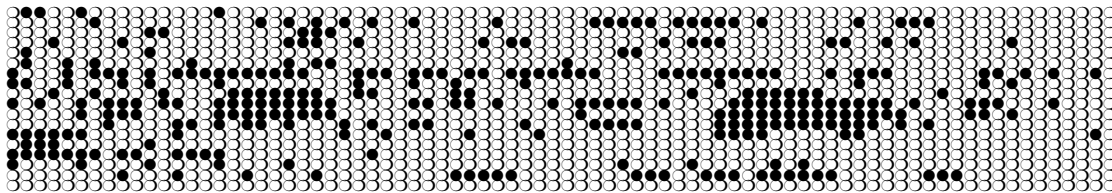
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	structure	information	computerised cooperators	number of players
13.	Group	detailed info	0	5
14.	Group	detailed info	0	5
15.	Group	detailed info	0	5
16.	Group	detailed info	0	5
17.	Group	detailed info	0	5
18.	Group	detailed info	0	5
19.	Group	detailed info	0	5
20.	Circle	less info	0	14
21.	Circle	less info	0	18
22.	Circle	less info	0	18
23.	Circle	less info	0	18
24.	Circle	less info	0	14
25.	Circle	less info	5	13
26.	Circle	less info	5	10
27.	Circle	less info	5	13
28.	Circle	less info	5	10
29.	Circle	less info	5	13
30.	Circle	less info	5	13
31.	Circle	detailed info	0	18
32.	Circle	detailed info	0	18
33.	Circle	detailed info	0	18
34.	Circle	detailed info	0	18
35.	Circle	detailed info	0	18

B Raw data

In the following graphs each line represents the actions of a player from period 1 to period 80. Cooperation is shown as ●, non cooperation as ○. Neighbouring lines correspond to neighbouring players in the experiment. In all treatments without computerised cooperators (sections B.1 to B.4) the last line of each block of lines is in circles always a neighbour of the first line of the same block. In these sections the display of circles is always rotated such that least cooperative players are found in the first and the last lines.

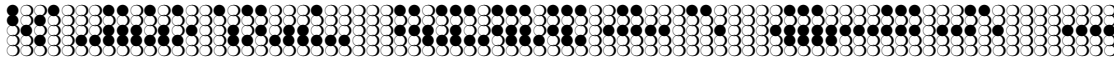
B.1 Circle treatment with detailed information



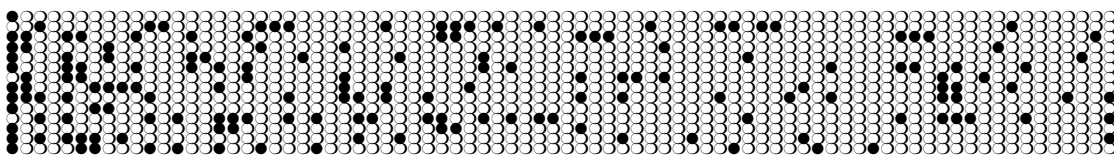
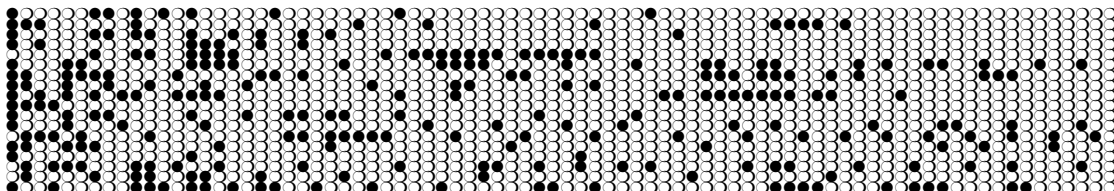
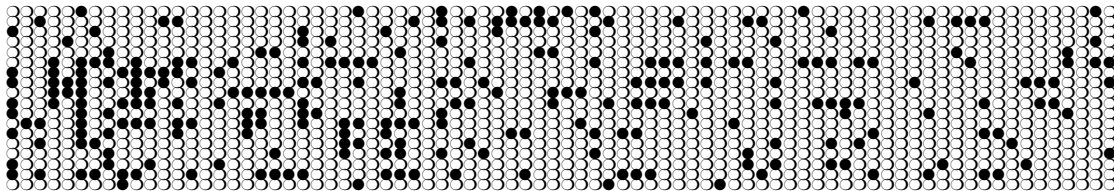
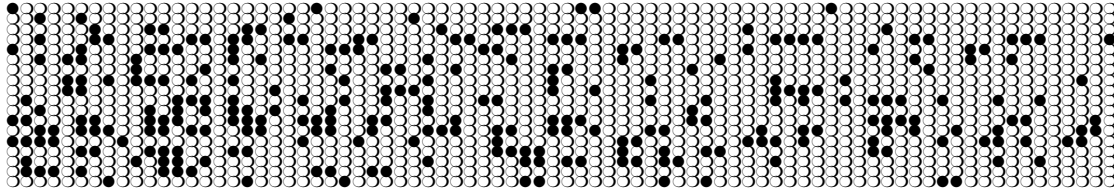
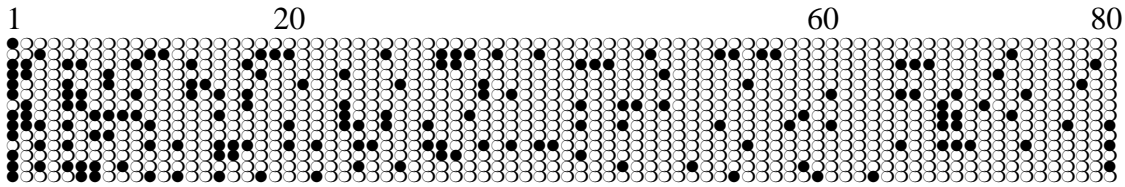


B.2 Group treatment with detailed information

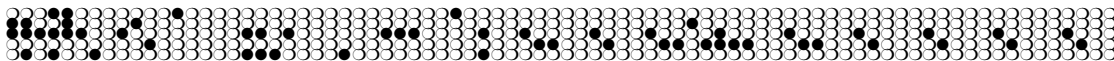
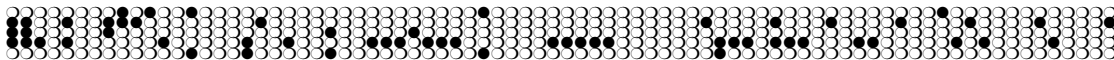
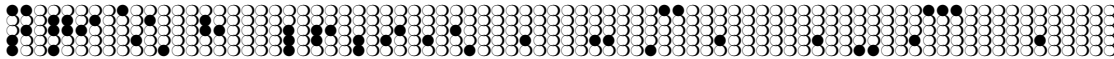
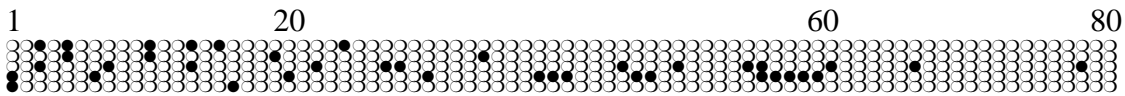


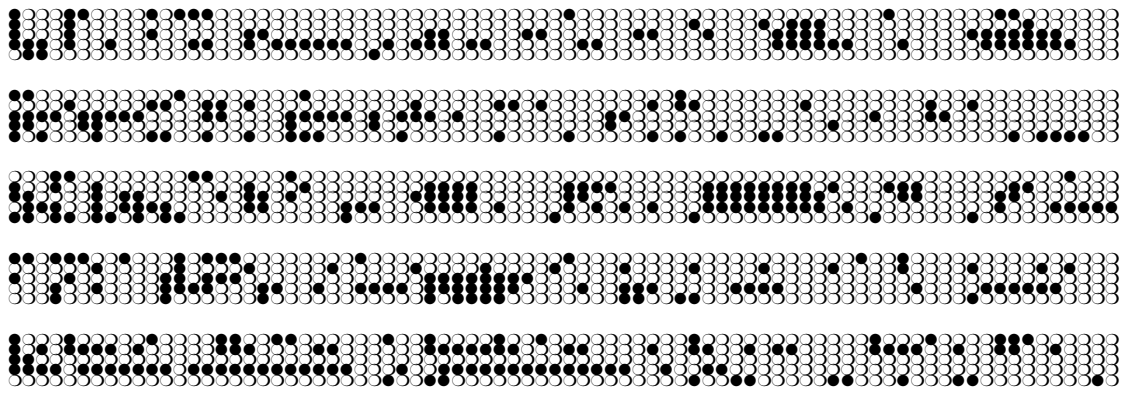


B.3 Circle treatment with less information



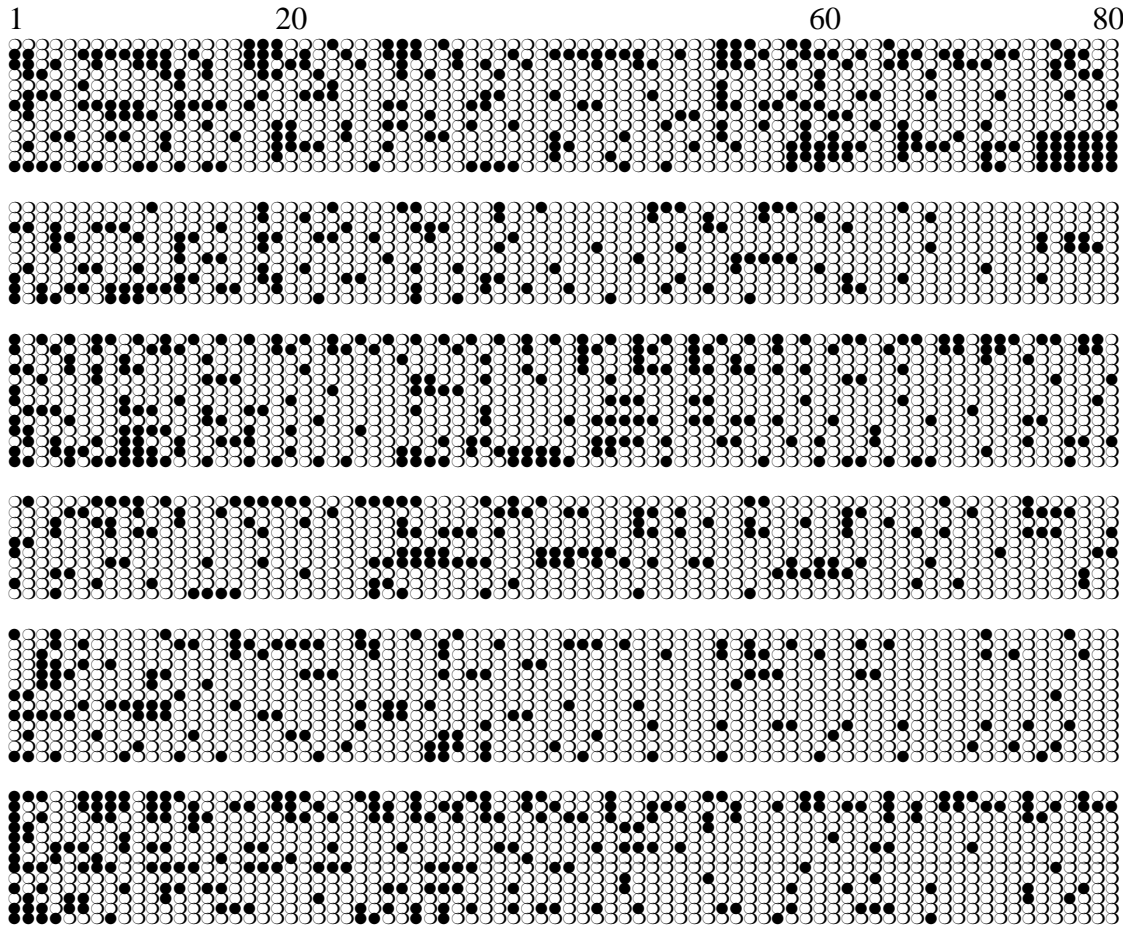
B.4 Group treatment with less information





B.5 Seeded circle with less information

In the display of the circles the five computerised cooperators are not displayed. Their location is on top of the first line and below the last line of each block. The two top lines and the two bottom lines of each block are, hence, immediate neighbours of computerised cooperators.



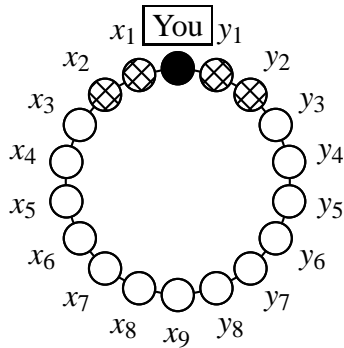
C Instructions of the Experiment

Please sit down and read the following instructions. It is important that you read them attentively. A good understanding of the game is a prerequisite of your success.

After having read the instructions you will continue with a little quiz on the computer screen. There you will be asked questions that will be easy to answer once you have read the instructions.

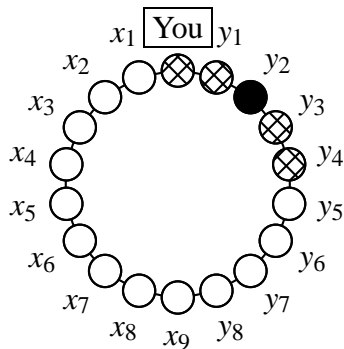
You may take notes but you may not talk to each other.

C.1 The structure of the neighbourhood



Your gain depends on your decision and on the decision of your two neighbours to the left and your two neighbours to the right. These four neighbours remain the same during the course of the experiment. You are connected through the computer with these neighbours. We will not tell who these neighbours are. Similarly your neighbours will not be told who you are.

In the diagram on the right side your four neighbours are shown cross-hatched.



Also your neighbours have neighbours. E.g. the neighbours of y_2 are players y_4 , y_3 , y_1 and you.

C.2 Rounds

In this experiment you play several rounds. In each round you take a decision. Depending on your decision and on the decision of your neighbours you receive points that will be converted to DM at the end of the experiment.

C.3 Decision

In each round you choose among two decisions. You choose A or B. Your gain depends on what you have chosen and on how many of your neighbours have chosen A or B.

This relation between choices and gains is the same for all participants.

It will be shown on the screen in the form of a table.

	Your neighbours play. . .
You play A	. . . Your gain . . .
You play B	

All players choose simultaneously, without knowing the decision of the others.

When all players have made their decision we continue with the next round.

C.4 Information after each round

In each round you receive information about your gain. Additionally you receive information about the decision of your neighbours and their gain.

Round	Your Decision	Your Gain	Decisions and gain in your neighbourhood, ordered by gain
...

In each row you obtain information about one round. You find your decision and your gain the second and the third column.

On the right side we show for each of your neighbours the decision of the neighbour and the obtained gain. The ordering of neighbours in this column depends on the gain in this period. First comes the neighbour with the highest gain, then the one whose gain was second, etc.. This implies that in each period a different person can be the first in the right column.

C.5 Quiz

Please answer now the questions from the quiz on the computer screen. If you are unsure how to answer a question, please consult your instructions.

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