

**Notation** The following notation is used interchangeably to denote derivatives:

Leibniz's (1675) notation:  $\frac{df}{dx}$  and  $\frac{d^2f}{dx^2}$  for the first and second derivative.

Lagrange's (1772) notation:  $f'$ ,  $f''$  for the first and second derivative. This notation is shorter but assumes that we know that derivatives are taken with respect to  $x$ .

Taking the first derivative of  $f'$  we obtain the second derivative  $f''$ .

Note that we write  $f$  and  $f(x)$  interchangeably. The former is shorter, but it assumes that we know that  $f$  is actually a function of  $x$ .

### Optimisation

- Minimum:  $f'(x) = 0$  and  $f''(x) > 0$
- Maximum:  $f'(x) = 0$  and  $f''(x) < 0$

**Differentiation rules** In the following  $a$  is a constant and  $u$ ,  $v$  and  $f$  are functions of  $x$ .

$f =$	$f' =$
$a$	$0$
$au$	$au'$
$u + v$	$u' + v'$
$uv$	$u'v + uv'$
$u(v)$	$u'(v)v'$
$\frac{u}{v}$	$\frac{u'v - v'u}{v^2}$
$x^n$	$nx^{n-1}$
$e^x$	$e^x$
$\ln x$	$1/x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$