Auctions

The Coase theorem

- without transaction costs, all government allocations are equally efficient, since parties will bargain to correct any externality.
- with transaction costs, government may minimize inefficiency by allocating property initially to the party assigning it the greatest utility.

private valuation: Bidders know their own x_i

uncertainty about $x_{j\neq i}$.

Example: consumption goods that are well known, that are not retraded.

Model: the vector of valuations x follows some distribution $F(x) : \mathfrak{R}^N \longmapsto \mathfrak{R}^N$

We will often assume the following:

- distribution is **independent**, $F_i(x_i|x_{j\neq i}) = F_i(x_i)$
- distribution is **identical** $F_i(x_i|x_{j\neq i}) = F(x_i|x_{j\neq i})$

example: uniform distribution over $[0,\omega]$, $F(x_i) = x_i/\omega$.

common valuation (informational externality): Bidder **does not know** the own valuation x_i, only knows a **signal** s_i

valuation x_i is a function of all signals $x_i = f(s_1, s_2, \ldots, s_i \ldots)$.

- special case: $x_i = \frac{1}{n}\sum s_i$

  all valuations are the same ex-post, expected valuations are not the same.

- other bidders $j \neq i$ have some information which is essential for bidder i.

- note: **valuations are typically not independent**

allocative externalities: valuation depends on who obtains a certain item.

multiunit auction: more than one good is sold

- homogeneous goods
- heterogeneous goods

multiunit demand: bidders have demand for more than one unit.

- complements (bidding for a knife and a fork)
- substitutes (decreasing marginal utility)

Goals

efficiency putting the licenses into the hands of those who value them the most (Vice President Al Gore)

competition (low prices on the end user market)

revenue

other goals

- Information aggregation
- Discovery of prices
- Fairness and transparency
- Administrative efficiency.

Auction formats for single-object auctions

- first-price sealed-bid auction
- second-price sealed-bid auction
- Dutch auction
- English auction (75% of all auctions, Cassidy)

Implementation of FCC auction

- 60 MHz 3G spectrum auctions in 2000

How can we possibly sell a frequency spectrum?

comparative hearing regulator decides, hardly objective, inefficient, low revenue

lottery with potential resale many will apply for the lottery, shifts the allocation problem to a different market (negotiating with lottery winners), but not necessarily more efficient

auction market

- Multiple item auction
- series of rounds
- in each round bidders make sealed bids for several licenses
- at the end of each round: standing high bid is posted
- bid-increment in next round (5%-10% to standing high bid)
- activity rule

FCC auctions in Europe

per capita revenue of 60 MHz 3G spectrum auctions in 2000
(ordered according to time of the auction):

country	2G	3G	€/head	endog. market	MHz	abstract	all bids visible	multi-round
UK	4	5	650		$3 \times 10, 2 \times 15^{ee}$		v	m
Netherlands	5	5	170		$2 \times 15, 3 \times 10$			m
Germany	4	6	620	*	$12 \times 5\ ^{2-3}$	a		m
Italy	4	5	210		5×10+5^{e}	a		m
Austria	4	6	105	*	$12 \times 5\ ^{2-3}$	a		m
Switzerland	3	4	20		4×15MHz			m
Belgium	3	4	45		4×15MHz			m
Denmark	4	4	95		4×15MHz			4^{th}-price
Spain			beauty contest					
Norway			beauty contest					
Sweden			beauty contest					
Finland			beauty contest					
France			beauty contest					

^{e}: 5 extra MHz for new entrants only

^{ee}: one 15MHz license for a newcomer only

$^{2-3}$: activity for 2-3 lots required

(from van Damme, EER, 2002)

- note: in the netherlands 5 licenses are sold to 5 incumbents: can an entryant expect to win?

Optimal bids in a second price auction

Proposition 1 *In a second price auction it is a weakly dominant strategy to bid according to* $\beta^{\mathrm{II}}(x) = x$.

The Revenue Equivalence Principle

standard auction the person who bids the highest amount wins the object

independent and identical distribution of valuations

risk neutral bidders maximise only expected payoff

Proposition 2 *. . . Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of the a bidder with value zero is zero, yields the same expected revenue to the seller.*

Note: ex-post revenue may differ (and often does)!

Proposition 3 *The symmetric bidding stratety in the equilibrium of a first-price auction is*

$$\beta^{\mathrm{I}}(x) = E[Y_1^{(n-1)} | Y_1^{(n-1)} < x]$$

where $Y_1^{(n-1)}$ *is the max of* $n-1$ *independent draws of* X_i*.*