SONDERFORSCHUNGSBEREICH 504

Rationalitätskonzepte,
Entscheidungsverhalten und
ökonomische Modellierung

No. 04-32

The overbidding-myth and the underbidding-bias in first-price auctions

Oliver Kirchkamp*
and J. Philipp Reiß**

October 2004

Financial support from the Deutsche Forschungsgemeinschaft, SFB 504, at the University of Mannheim, is gratefully acknowledged.

*Sonderforschungsbereich 504, email: oliver@kirchkamp.de

**University of Magdeburg, Faculty of Economics and Management May 2004, email: reiss@ww.uni-magdeburg.de

Universität Mannheim
L 13,15
68131 Mannheim

The overbidding-myth and the underbidding-bias in first-price auctions*

Oliver Kirchkamp[†] Philipp Reiss

November 11, 2004

Abstract

First-price auction experiments find often substantial overbidding which is typically related to risk aversion. We introduce a model where some bidders use simple linear bids. As with risk aversion this leads to overbidding if valuations are high, but in contrast to risk aversion the model predicts underbidding if valuations are low.

We test this model with the help of experiments, compare bidding in first-price and second-price auctions and study revenue under different treatments.

We conclude that at least part of the commonly observed overbidding is an artefact of experimental setups which rule out underbidding. Simple linear bids seem to fit observations better.

Keywords: Auction, Experiment, Overbidding, Underbidding, Risk-Aversion.

(JEL C92, D44)

1 Introduction

First-price sealed bid auctions are a common institution and equilibrium bidding functions can often be derived in a straightforward way. Bidding behaviour has also been studied in the lab and found to deviate systematically from equilibrium bids. Cox, Smith, and Walker (1983, 1985, 1988) are among the first to report on a large dataset with first-price auctions. Figure 1 shows data from one of their experiments. Participants repeatedly play a first-price auction with a fixed number of bidders. For each participant valuations are drawn from a uniform distribution with support $[0, 1]$. The figure shows the valuations that were drawn for a specific bidder on the horizontal and the bids on the vertical axis.

The line $b = \frac{3}{4}x$ shows the equilibrium bidding function for this auction. We see that most bids are above the equilibrium bidding function which we call overbidding. Overbidding in first-price auctions has been found for most participants in the experiments

*Financial support from the Deutsche Forschungsgemeinschaft through SFB 504 is gratefully acknowledged.

[†]University of Mannheim, SFB 504, L 13, 15, D-68131 Mannheim, email oliver@kirchkamp.de

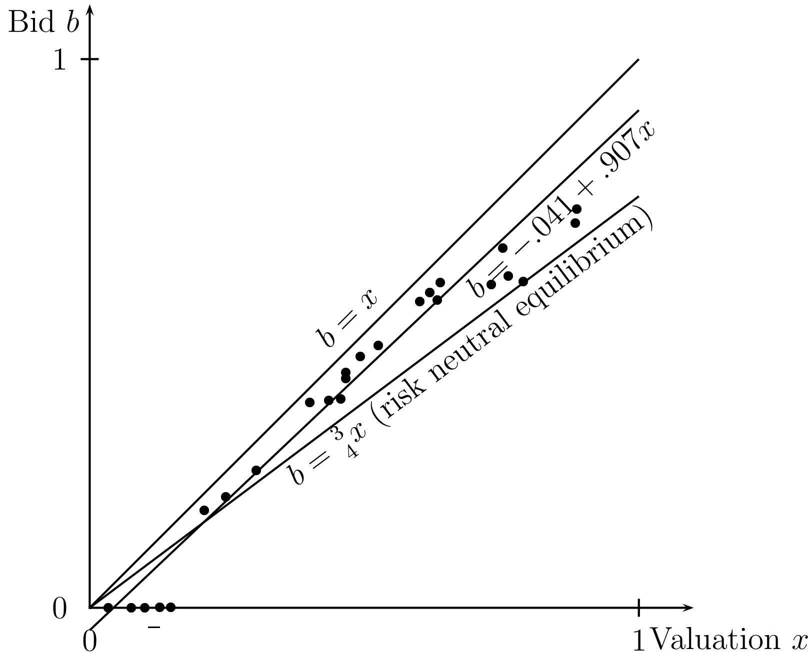


Figure 1: An example from an experiment by Cox, Smith, and Walker (1988)
(page 84, figure 8, series 4, exp. 3, n=4, subject 2)

by Cox, Smith, and Walker (1983, 1985, 1988) and has been replicated in many other studies since then.

A standard explanation for overbidding in first-price auctions is risk aversion. We can assume specific functional forms for utility functions, e.g. constant relative risk aversion (CRRA) which allows for steeper equilibrium bidding functions and, thus, a better approximation of actual bidding behaviour.

However, risk aversion does not explain all deviations from risk neutral equilibrium bidding. Cox, Smith, and Walker (1985) find overbidding even in experiments where subjects are payed in lottery tickets—i.e. where a subject with von Neumann-Morgenstern utility must behave in a risk neutral way. In experiments with third-price auctions, where risk averse bidders should bid less and not more than the risk neutral equilibrium bid, Kagel and Levin (1993) find overbidding and no underbidding.

Let us have a look at figure 1 again. For low valuations many bids are below, not above, the equilibrium bid. This is neither consistent with risk neutral utility functions nor with CRRA or variants thereof. The example shown in figure 1 is not the only case. Cox, Smith, and Walker (1988) find that when they approximate bids with the help of linear bidding functions these functions do not always intersect with the origin. Positive intercepts of the bidding function with the b-axis are rationalised through some 'utility of winning' in their CRRAM* approach, but it is difficult to find an explanation for negative intercepts.

Often in auction experiments underbidding is ruled out through the experimental setup. For the experimenter it is tempting to choose zero as a lower boundary of the interval of valuations. Typically participants are explicitely prevented from entering negative bids or implicitely lead to the assumption that bids must be positive. In either case the experimental design introduces a bias against underbidding. Of course, risk neutral or even risk averse bidders that follow CRRA would never make negative bids in this situation, so choosing valuations from an interval $[0, \bar{\omega}]$ looks like a innocent simplification that makes it easier to explain the rules of the experiment.

When experimenters do not choose zero as the lowest possible valuation, as e.g. Ivanova-Stenzel and Sonsino (2004) and Güth, Ivanova-Stenzel, and Wolfstetter (2004) do, they sometimes find underbidding. Ivanova-Stenzel and Sonsino (2004) observe that 7.4% of the bids in their first-price auction are below the lowest possible valuation. They attribute underbidding to an attempt to obtain a low price.

Our aim in this paper is to find an explanation that is compatible with the observations mentioned above. In the next section we will introduce a model where all or some bidders are boundedly rational and use particularly simple linear bids. As with risk aversion this leads to overbidding for high valuations, but in contrast to risk aversion simple linear bids predict underbidding for low valuations. In section 3 we will test this model with the help of experiments. We will compare bidding in first-price and second-price auctions and study revenue under different treatments.

Anticipating our result we will find that at least part of the commonly observed overbidding is an artefact of experimental setups which rule out underbidding. Simple linear bids explain both the overbidding for high valuations as well as the underbidding for low valuations.

2 Model

In the following we will concentrate on a first-price sealed-bid auction with two bidders. Sometimes we will refer to the case of a second-price sealed-bid auction for comparison. In sections 2.1, 2.2, and 2.4 we will derive optimal bidding functions for different contexts. Section 2.5 will introduce the parameters for our experiment.

2.1 Equilibrium Bids

Deriving the Bayesian Nash Equilibrium for the first-price sealed bid auction is standard and is repeated here to make the reader familar with the notation. Let us for simplicity restrict attention to the case where valuations are distributed uniformly over $[0, 1]$. Assume that utility functions have the form $u(x) = x^r$ where r is a parameter that measures attitude towards risk. A risk neutral individual would be described by $r = 1$, a risk averse individual would be characterised by $r < 1$. While there are some individuals who behave risk loving in the laboratory, the majority does not. We will, hence, concentrate on the case $r \in (0, 1]$. To find a Bayesian Nash equilibrium let us assume that there is a symmetric increasing bidding function $\gamma(x)$ which is invertible. In equilibrium all bidders bid according to γ. In a situation with two bidders we have to show that if bidder 2 follows γ then it is also a best reply for bidder 1 to follow γ. Bidder 1 with true valuation x will make a bid b. Since γ is invertible we can find a valuation z such that $b = \gamma(z)$. Bidder 1 wins the auction when the valuation of the other bidder is smaller than z. The probability of this event is $G(z) = z$. Bidder 1 chooses z to maximise $\mathrm{EU} = G(z) \cdot u(x - \gamma(z))$ which

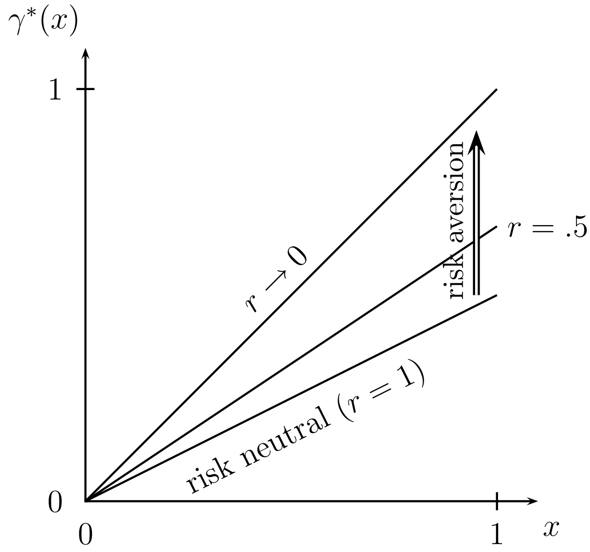


Figure 2: Bids in the Bayesian Nash Equilibrium

yields as a first order condition

$$(x - \gamma(z))^r - r \cdot z \cdot (x - \gamma(z))^{r-1}\gamma'(z) = 0 \tag{1}$$

In the symmetric equilibrium we will have $z = x$, hence,

$$(x - \gamma(x))^{r-1} \cdot (\gamma(x) + x \cdot (r\gamma'(x) - 1)) = 0 \tag{2}$$

For $r \in (0, 1]$ there are several solutions for this differential equation. It is easy to see that in equilibrium $\gamma(0) = 0$ which yields the unique solution

$$\gamma^*(x) = \frac{x}{1 + r} \tag{3}$$

The second derivative $\partial^2\mathrm{EU}/\partial z^2 = -(rx/(1+r))^{r-1}$ is negative, so we have indeed found a maximum. If valuations are not drawn from the interval $[0, 1]$ but from $[\underline{\omega}, \overline{\omega}]$ one finds similarly that the equilibrium bid is

$$\gamma^*(x) - \underline{\omega} = \frac{x - \underline{\omega}}{1 + r} \tag{4}$$

Figure 2 shows three examples. In a world where bidders are risk neutral ($r = 1$) they will all bid according to $b = x/2$. When bidders are more risk averse ($r < 1$) they will bid more. In the limit, if bidders are inifinitely risk averse ($r \to 0$), they will make bids equal to their valuation.

For second-price sealed-bid auctions a standard argument shows that it is always a weakly dominant strategy to make a bid equal to the own valuation.

$$\gamma^*(x) = x \tag{5}$$

This holds independently of the risk aversion.

2.2 Linear bidding functions

We have seen above that in the Bayesian Nash Equilibrium bidders in the first-price auction make bids which are smaller than their valuation, they 'shade their bids'. Indeed, many participants in our experiment report in the questionnaires which they complete at the end of the experiment that they bid 'a sufficient amount less' than their valuation to make a profit. In the Bayesian Nash equilibrium this shading is an amount that increases with the valuation. However, changing the amount of shading and, in particular, determining the right amount of the change, might be a difficult problem.

Some comments from participants in our experiment seem to indicate that it is easier to bid an amount that is just the own valuation shaded by a constant amount. Shading by a constant amount can also be related to satisficing behaviour. A bidder who wants to gain a certain amount when winning the auction must bid the own valuation minus this amount. Finally, shading by a constant amount can also be interpreted as a simple rule given to a bidding agent. If first a principal has to define a bidding rule and then the agent who follows this rule learns the valuation, then it might be simpler for the principal to require a fixed amount that the agent must gain from each trade.

Let consider a population of boundedly rational bidders which all bid according to a bidding function $\bar{\gamma}$ of the following structure:

$$\bar{\gamma}_i(x) = \alpha_i + \beta x \tag{6}$$

Let us assume that the parameter β is exogeneously given and the same for all bidders. We will mainly be interested in the case where bidders choose a bid that is just the valuation shaded by a constant amount, i.e. $\beta = 1$. This is what we will call simple linear bids in the following. In the Bayesian Nash Equilibrium in equation (3) we had $\beta = 1/(1+r)$.

While β is fixed for all bidders we consider a situation where each bidder determines the constant shading α_i individually. What is, given the exogenous parameter β, and given one's opponent's j choice of α_j, the best reply? Given two valuations x_i and x_j of two bidders, bidder i wins if his opponent's valuation $x_j < x_i + (\alpha_i - \alpha_j)/\beta$. Let us first consider the case $\alpha_i < \alpha_j$. Then the expected utility for a risk neutral bidder is

$$\mathrm{EU}_1 = \int_{\frac{\alpha_j-\alpha_i}{\beta}}^{1} \int_0^{x_i - \frac{\alpha_j-\alpha_i}{\beta}} x_i - (\alpha_i + \beta x_i) dx_j dx_i \tag{7}$$

The first order condition yields

$$\alpha_i^* = \frac{\alpha_j + \beta - 2\beta^2}{1 + 2\beta} \tag{8}$$

This condition is fulfilled for both players if

$$\alpha_i = \alpha_j = \frac{1}{2} - \beta \tag{9}$$

In the same way we can study the case $\alpha_i > \alpha_j$ which yields the same condition.

We should note that the risk neutral Bayesian Nash Equilibrium is a special case ($\alpha = 0$, $\beta = 1/2$) of equation (9).

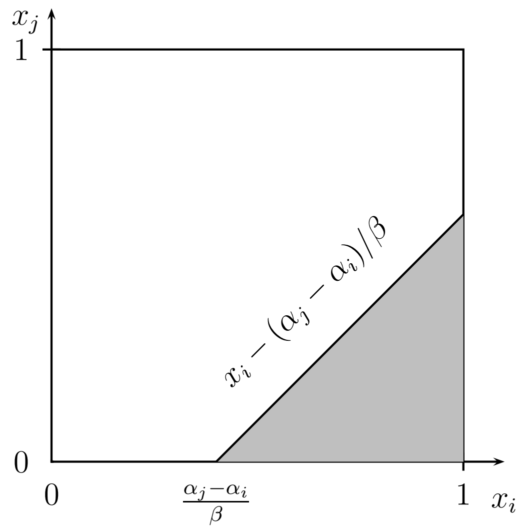


Figure 3: Region where bidder i wins if bids follow equation (6) and $\alpha_i < \alpha_j$

Equation (7) can be gerneralised to also account for risk aversion:

$$\mathrm{EU}_2 = \int_{\frac{\alpha_j-\alpha_i}{\beta}}^{1} \int_{0}^{x_i - \frac{\alpha_j-\alpha_i}{\beta}} (x_i - (\alpha_i + \beta x_i))^r \, dx_j dx_i \tag{10}$$

We can follow the same steps as above, however, for general β it is not possible to obtain a closed form solution for α_i as we could in equation (9). For the simple linear case $\beta = 1$, which we find particularly interesting, it is staightforward to find the equilibrium

$$\alpha = -\frac{r}{2} \tag{11}$$

Risk averse bidders who are restricted bid their valuation minus a constant amount will in equilibrium choose this constant to be $r/2$.

For the case of the second-price sealed-bid auction we see that the weakly dominant bidding strategy from equation (5) already has the simple structure of equation (6) with $\alpha_i = 0$ and $\beta_i = 1$. A bidder who is restricted to choose a bid that is the valuation shaded by a constant amount will, thus, not deviate from the weakly dominant bid.

2.3 Revenue comparison

With risk neutral bidders in the Bayesian Nash equilibrium the well known revenue equivalence theorem holds. For a wide range of auctions the expected revenue is the same. In particular the expected revenue in the first-price and in the second-price sealed bid auction do not differ.

It is easy to see that with risk averse bidders in the Bayesian Nash equilibrium the first-price auction generates a higher revenue. From equation (3) we see that bids increase with risk aversion in the first-price auction. If bidders follow the bidding strategy from equation (3) the expected revenue in the first-price auction is

$$R^{\mathrm{I}} = 2\int_0^1 \int_x^1 \frac{y}{1+r}\,dy\,dx = \frac{2}{3+3r} \tag{12}$$

In the second-price auction it is still a dominant strategy to bid the own valuation. Expected revenue in the second-price auction is, hence,

$$R^{\mathrm{II}} = 2\int_0^1 \int_x^1 x\,dy\,dx = \frac{1}{3} \tag{13}$$

Thus, $R^{\mathrm{II}} < R^{\mathrm{I}}$ for $r < 1$.

If bidders in the first-price auction no longer follow the Bayesian Nash equilibrium but, instead, follow the linear bidding function from equation (6) then the expected revenue is

$$\bar{R} = 2\int_0^1 \int_x^1 \beta y + \alpha\,dy\,dx = \alpha + \frac{2\beta}{3} \tag{14}$$

We see that $R^{\mathrm{II}} < \bar{R}$ iff $\alpha > (1-2\beta)/3$. For the case of risk averse linear bidders that we study in equations (10) and (11) we find that $R^{\mathrm{II}} < \bar{R}$ iff $r < \frac{2}{3}$.

Thus, similar to the Bayesian Nash equilibrium more risk aversion increases the revenue obtained in the first-price auction. However, a higher degree of risk aversion is needed to outperform the second-price auction than in the Bayesian Nash equilibrium. In section 3.2 we will compare revenue of the first-price auction and the second-price auction in the experiment.

2.4 Optimising against a linear bidder

To complete the discussion of linear bidding functions let us consider the case of a rational bidder who knows that the opponent makes a bid $\bar{\gamma}(x_j)$ according to equation (6) where parameters α_j and β of the opponent are known but where the valuation x_j of the opponent is unknown. What is the best response $\hat{\gamma}$ in a first-price auction against such a linear bidder? Here we can not simplify equation (1) by assuming symmetry ($x = z$) as we did in the derivation of equation (2). Instead, we have to substitute equation (6) into (1). Solving the first order condition we obtain

$$z = \frac{x-\alpha}{(1+r)\beta} \tag{15}$$

which, using (6), yields

$$\hat{\gamma}(x) = \frac{x + r\alpha}{1+r} \tag{16}$$

To make sure that the step from equation (1) to (15) is legitimate we need that the inverse of $\bar{\gamma}$ is in $[0,1]$ for all values of $\hat{\gamma}(x)$ with $x \in [0,1]$. That requires that $\alpha < 0$ and $\beta > (1-\alpha)/(1+r)$. If $\beta = 1$ and α is chosen according to equation (9) this is always the case. Thus, in the case of simple linear bids the requirement is satisfied. The requirement is even fulfilled for a broader range of parameters. E.g. if $r = 1$ then $\beta > 1/2$ is sufficent.

Note that from equation 16 follows that if some boundedly rational bidders use simple linear bids then also rational bidders will use a linear bid with a negative intercept, i.e. we will have underbidding for low valuations.

Treatment		$[\underline{\omega}, \overline{\omega}]$	restriction of bids	auction
-25	$-$	$[-25, 25]$	—	first-price
0	$-$	$[0, 50]$	—	first-price
0	$+$	$[0, 50]$	only positive bids	first-price
25	$-$	$[25, 75]$	—	first-price
50	$-$	$[50, 100]$	—	first-price
50	$+$	$[50, 100]$	only positive bids	first-price
50	$+II$	$[50, 100]$	only positive bids	second-price

Table 1: Treatments

2.5 Experimental setup

The purpose of the experiment to twofold: We want to find out how far simple linear bids are consistent with actual behaviour and we want to check to what extent the existing experimental evidence on first-price auctions is an artefact of the design. To distinguish simple linear bids from risk-averse bids we should be able to observe bids also for low valuations in a reliable way since particularly in this region predictions of risk-aversion and simple linear bids differ. It has been argued that for low valuations bidders seldom win the auction and therefore have little chance to gain experience. To allow bidders to gain as much experience as possible we use a setup with two bidders only. Furthermore we use the strategy method and play in each round five independent auctions which increases the chance to get feedback further.

So far in this section we assumed that bidders' valuations are distributed uniformly over an interval $[0, 1]$. This simplifies the analysis and is still not such a severe restriction since all results apply in a similar way to affine transformations of valuations and bids. I.e. all that we have said above can be generalised to the case where bidders' valuations are distributed uniformly over an interval $[\underline{\omega}, \overline{\omega}]$.

In our experiment we vary three parameters:

- We will study different ranges for the valuation. In some cases the lowest possible valuation $\underline{\omega}$ is zero, in others $\underline{\omega}$ is positive, and in others it is negative. In all cases the highest possible valuation $\overline{\omega} = \underline{\omega} + 50$. Valuations are always uniformly distributed over this range.

- We will sometimes allow for negative bids and sometimes we will not allow for negative bids.

- Most of our auctions will be first-price sealed-bid auctions. We have run also second-price auctions as a control.

Table 1 gives an overview. For each bidder the density of a valuation x is $g(x) = 1/50$ and the distribution

$$G(x) = \begin{cases} 1 & \text{if } x > \overline{\omega} \\ (x - \underline{\omega})/50 & \text{if } x \in [\underline{\omega}, \overline{\omega}] \\ 0 & \text{otherwise} \end{cases}$$

Experiments were conducted between 12/2003 and 12/2004 in the experimental laboratory of the SFB 504 in Mannheim. 276 subjects participated in these experiments. A

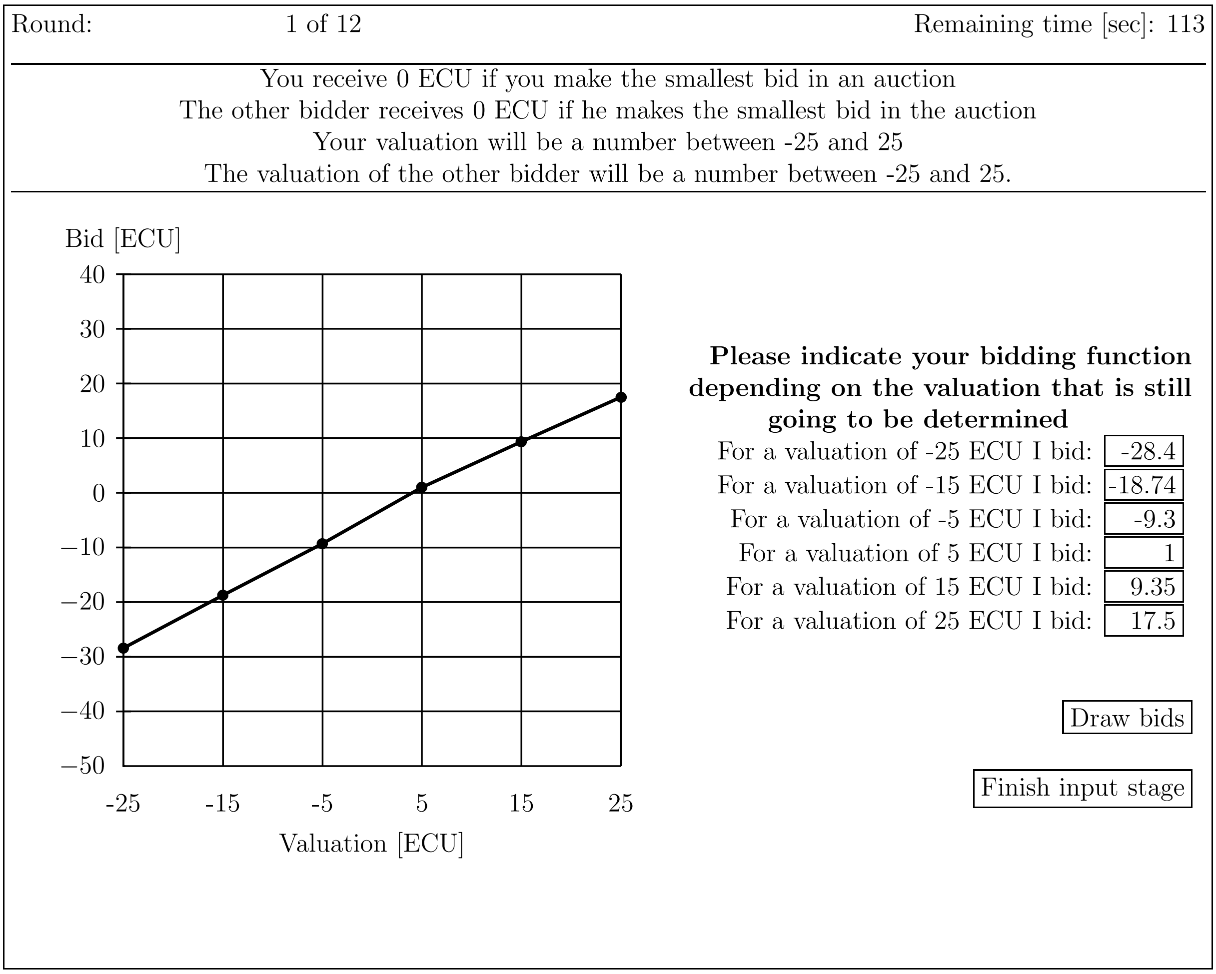


Figure 4: A typical input screen in the experiment (translated into english)

detailed list of the treatments is given in appendix A, instructions are shown in appendix B. The software we used was z-Tree (Fischbacher (1999)).

We used the strategy method to elicit bids for several valuations in each round. A typical screen during the experiment is shown in figure 4 (translated into english). In each round participants enter bids for six valuations which are equally spaced between $\underline{\omega}$ and $\overline{\omega}$. Bids for all other valuations will be later interpolated linearly. When all participants have determined their bidding functions we draw five random and independent valuations for each participant. Each of these five random draws corresponds to an auction for which the winner is determined and the gain of each player is calculated. The sum of the gain of these five auctions determines the total gain from this round. A typical feedback screen is shown in figure 5. This procedure is repeated for 12 rounds. After that participants complete a small questionnaire and are then payed in cash according to their gains in the experiment.

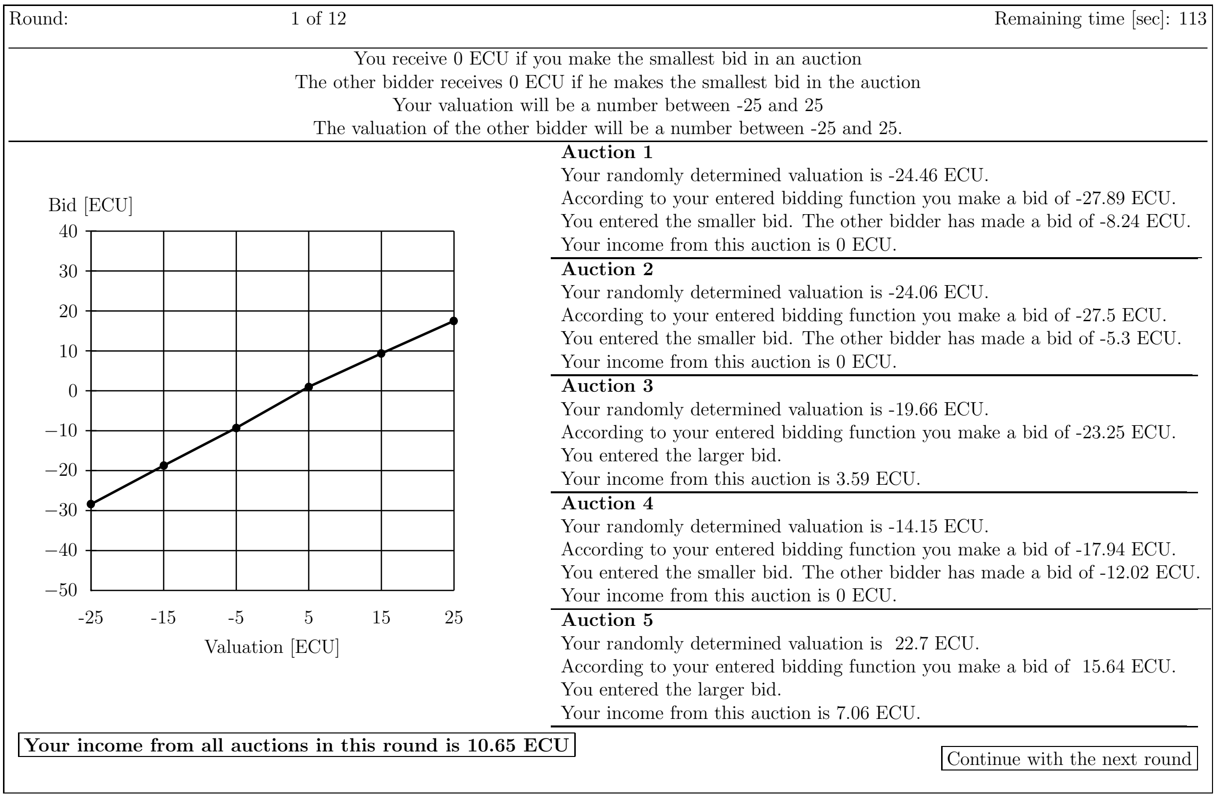


Figure 5: A typical feedback screen in the experiment (translated into english)

3 Results

3.1 Bids in the experiment

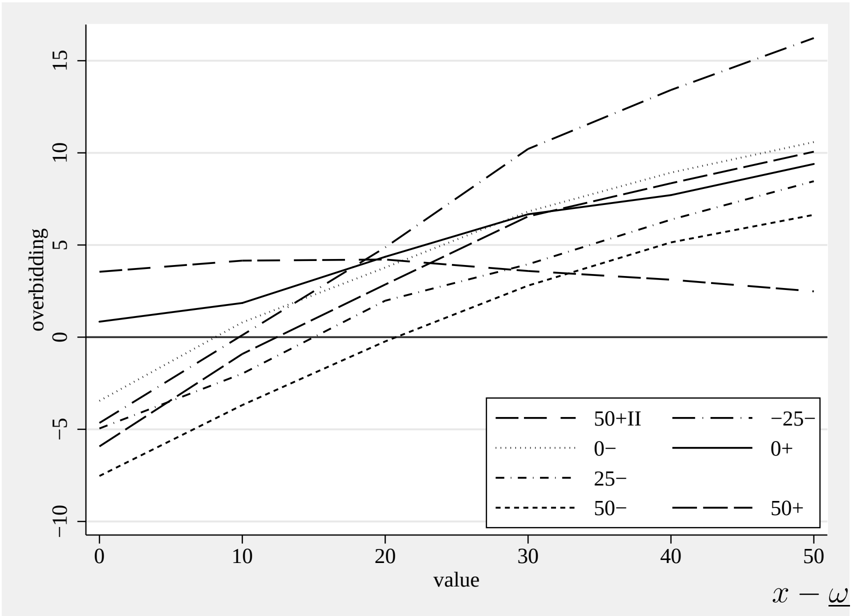
The average amount of overbidding $b(x) - \gamma^*(x)$ as a function of the valuation x, is shown in figure 6. Let us look at the $50 + II$ and the 0+ treatment first.

The $50 + II$ treatment is a second-price treatment where bidders have a weakly dominant bidding strategy: Bid the own valuation. We see that on average our bidders bid slightly more than this. The amount of overbidding is always positive and does not change much with the valuation. This is consistent with Kagel, Harstad, and Levin (1987) and Kagel and Levin (1993) who also find systematic overbidding in the second-price auction, though not in the English auction. They relate to overbidding in the second-price auction to “the illusion that it improves the probability of winning with no real cost to the bidder” (Kagel, Harstad, and Levin, 1987, p. 1299). Another possible explanation is that bidders have ‘utility of winning’ as in Cox, Smith, and Walker (1988).[1]

The 0+ treatment is the traditional first-price treatment. The lowest possible valuation is 0, and bids are restricted to be positive. We see that there is overbidding. The amount of overbidding increases with the valuation. This finding is consistent with risk-aversion and confirms findings from several previous experiments, starting with

[1]It is straightforward to include a constant ‘utility of winning’ into the model presented in section 2 and to obtain an even better fit of the experimental data in this and the other treatments.

$b(x) - \gamma^*(x)$



$x - \underline{\omega}$

Figure 6: Overbidding for different treatments
The figure shows averages for all participants. The first 6 periods from each session are discarded.

Cox, Roberson, and Smith (1982).

The other five treatments allow for underbidding and, indeed, show underbidding if the valuation is small. This is not consistent with risk-averse behaviour but it is consistent with the simple linear bids presented in section 2.

Before we study this in more detail let us first look at changes in bidding behaviour over time. To do that we assume that players follow a linear[2] bidding function as follows:

$$b(x) = \underline{\omega} + \alpha + \beta \cdot (x - \underline{\omega}) + u \tag{17}$$

where x is the bidder's valuation. In figure 7 we show estimated coefficients of this equation for each period and each treatment. Let us have a look at the right part of the figure: In the Bayesian Nash equilibrium we should have $\beta = 1/2$, with simple linear bids $\beta = 1$. From the figure we see that the estimated β is closer to 1 than to 1/2 (except in the 0+ treatment where underbidding is excluded by the experimental setup). In the right part of the figure we show the intercept of the bidding function α which should be zero in the Bayesian Nash equilibrium. Indeed, α is close to zero in the 0+ treatment, it is positive in the second-price treatment, but negative in all other first-price treatments.

Figure 8 shows how overbidding for the six valuations which are entered during the experiment and how overbidding reacts on the range of valuations. When the lower

[2]We have also estimated a quadratic bidding function with very similar results and small coefficients for the quadratic term. To be constistent with the models presented in sections 2 which are all linear we present here only results of the linear estimation.

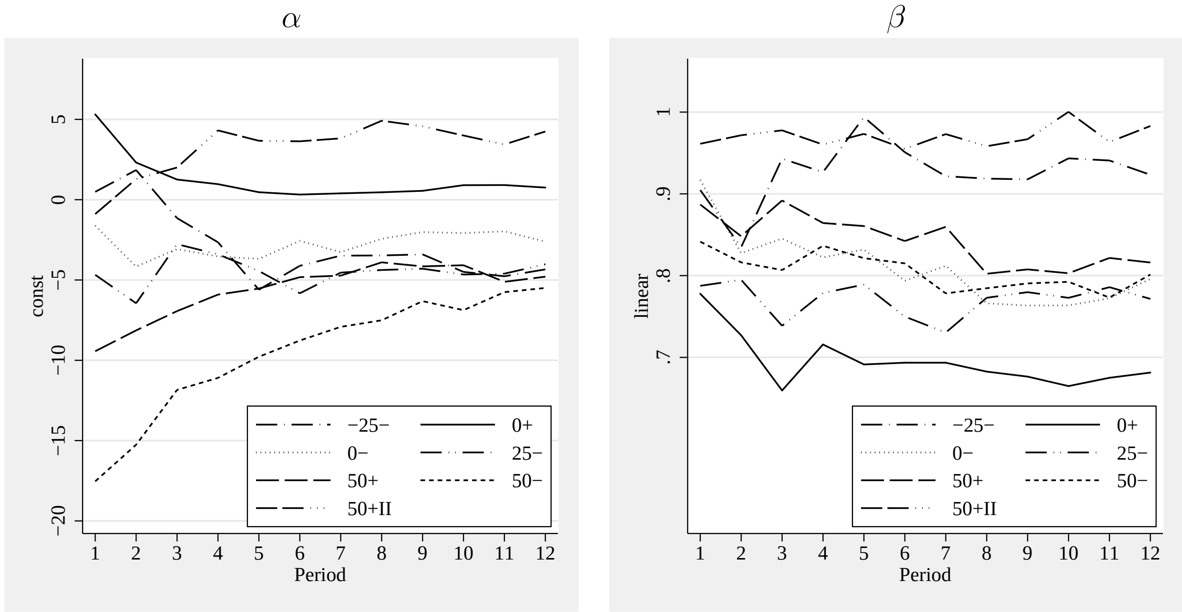


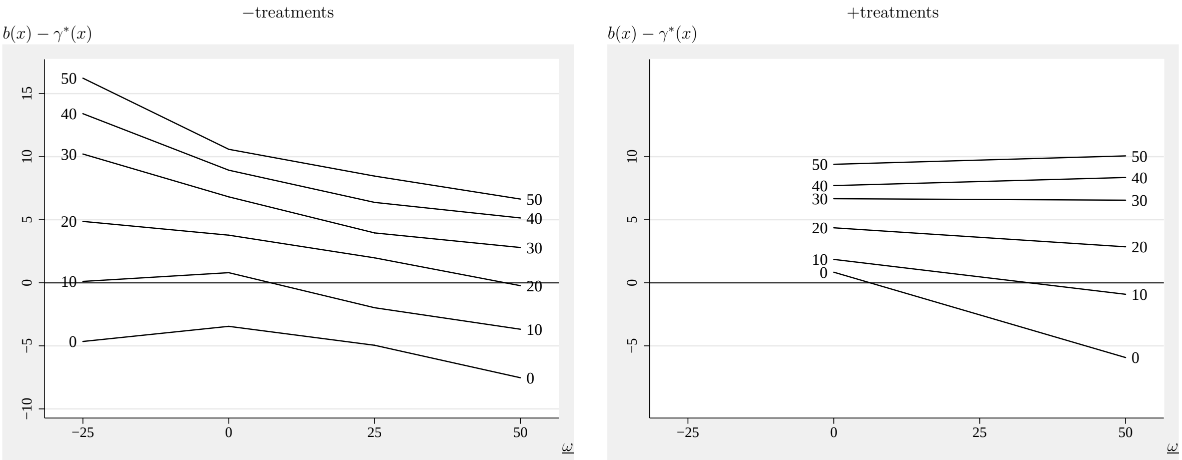
Figure 7: Coefficients of linear bidding functions over time
The graph shows for each treatment and for each period the estimated coefficients of equation (17).

boundary of possible valuations is exactly zero the degree of underbidding is smaller than with the other lower boundaries, even when participants have technically the possibility and even when negative bids are mentioned in the instructions. In all other cases we find underbidding with low valuations.

Table 2 shows mean overbidding for the highest and lowest valuation, $\underline{\omega}$ and $\overline{\omega}$, together with results from parametric and nonparametric tests. The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuation) and whether whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuation). In both cases mean bids are shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). As in other experiments with first-price auctions, we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. We also find underbidding for the smallest possible valuation $\underline{\omega}$ in all treatments where underbidding is possible, i.e. always, except in the 0+ treatment.

3.2 Revenue in the experiment

In section 3.1 we have found overbidding as well as underbidding as long as the design permits small bids. In the current section we investigate the net effect on revenue. Do the effects of overbidding and underbidding cancel out if small bids are possible or is overbidding still the dominating effect? The left graph in figure 9 shows average revenue for all first-price and for all second-price auctions over time. The Bayesian Nash equilibrium revenue is shown as a dotted horizontal line. After a few periods the pattern is fairly stable and in both types of auction above equilibrium. As in section 3.1 we will not use the first 6 periods from our analysis.



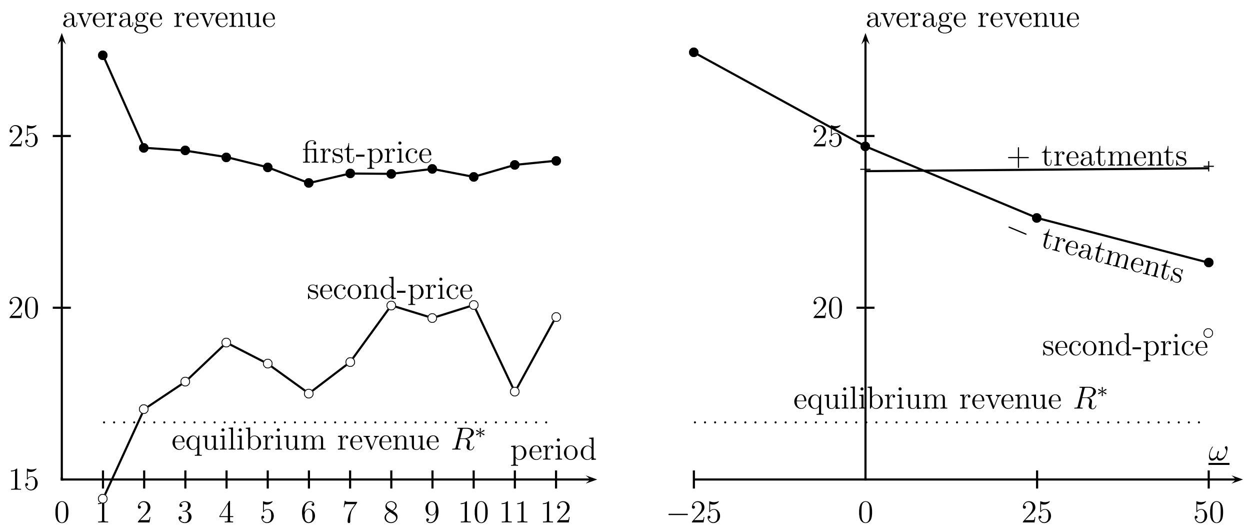
The figure shows averages of $b(x) - \gamma^*(x)$ with the first 6 periods of each session discarded. The left graph shows the four −treatments (-25-,0-,25-,50-) where participants are allowed to make negative bids, the right graph shows the two +treatments (0+,50+) where bids must be positive. The lowest possible valuation $\underline{\omega}$ for each treatment is shown on the horizontal axis.
Each graph shows six lines. The lowest line (denoted with 0 at the left and right end) shows average overbidding $b(x) - \gamma^*(x)$ for the lowest possible valuation $\underline{\omega}$. The next line (denoted with 10 at the left and right end) shows average overbidding for a valuation of $\underline{\omega} + 10$. The subsequent lines are denoted with 20 to 50 and show average overbidding for a valuation of $\underline{\omega} + 20$ up to a valuation of $\underline{\omega} + 50$ for the highest line.

Figure 8: Mean overbidding

		$b(\underline{\omega}) - \gamma^*(\underline{\omega})$				$b(\overline{\omega}) - \gamma^*(\overline{\omega})$			
treatment	n	mean	t	$P_{>t}$	P_{bin}	mean	t	$P_{>t}$	P_{bin}
-25 $-$	4	-4.653	-7.703	.0023	.0625	16.229	20.753	.0001	.0625
0 $-$	6	-3.453	-2.003	.0508	.0156	10.586	5.798	.0011	.0156
0 $+$	4	.841	2.009	.9309	1.000	9.397	14.154	.0004	.0625
25 $-$	3	-4.953	-5.574	.0154	.1250	8.461	4.176	.0264	.1250
50 $-$	4	-7.53	-4.643	.0094	.0625	6.639	2.806	.0338	.0625
50 $+$	11	-1.188	-.707	.2478	.5000	18.774	6.735	.0000	.0005
all first-price	26	-4.397	-6.489	.0000	.0001	10.304	13.831	.0000	.0000
second-price	6	3.549	3.98	.9947	.9844	27.485	36.084	.0000	.0156

Table 2: Overbidding for different treatments

The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuation) and whether whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuation). In both cases mean deviations from the risk neutral Bayesian Nash equilibrium bid are shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). As expected, we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. However, we also find underbidding for the smallest possible valuation $\underline{\omega}$ in all first-price treatments where underbidding is possible, i.e. always, except in the 0+ treatment and in the second-price auction.



The graph on the left side shows average revenue for all first-price and for all second-price auctions over time.
The graph on the right side shows how average revenue depends on $\underline{\omega}$ and on the type of the treatment. Averages are shown with the first 6 periods of each session discarded.

Figure 9: Revenue

The graph on the right-hand side of figure 9 shows how average revenue depends on $\underline{\omega}$ and on the type of the treatment. We see that all treatments yield excess revenue. However, excess revenue is higher in the first-price treatments than in our second-price treatment. In table 3 we look at at the precise value of the excess revenue, i.e. the average difference between the actual revenue R and the equilibrium value R^*. For each treatment n is the number of independent observations. We test whether $R > R^*$ (revenue in the experiment is higher than revenue in equilibrium). This is always the case and in most cases it is significant for the invididual treatment. Most importantly, average revenue for all first-price auctions is significantly higher than equilibrium revenue both with parametric and with non-parametric tests. Also in the second-price auction average revenue is significantly higher than in equilibrium. In the lower part of table 3 we compare revenue in the first-price and in the second-price auction. We find that average revenue is significantly higher in the first-price auction. The non-parametric test used here is a Wilcoxon rank-sum test.

4 Concluding remarks

In this paper we have presented a series of experiments in an attempt to better understand systematic deviations from equilibrium bids in first-price auctions.

A range of the classic evidence of first-price auction experiments finds that subjects bid more than the risk neutral equilibrium bid. The lack of risk-neutrality is a tempting explaination for this overbidding behaviour. However, risk aversion does not seem to explain everything. Cox, Smith, and Walker (1985) find overbidding even in experiments where subjects are payed in lottery tickets—i.e. where a subject with von Neumann-

treatment		n	$R - R^*$ mean	t	$P_{>t}$	P_{bin}
−25	−	4	10.793	33.758	.0001	.1250
0	−	6	8.049	7.958	.0005	.0312
0	+	4	7.298	34.935	.0001	.1250
25	−	3	5.974	3.708	.0657	.2500
50	−	4	4.667	4.064	.0269	.1250
50	+	5	7.375	13.068	.0002	.0625
all first-price		26	7.449	16.489	.0000	.0000
second-price		6	2.578	5.105	.0038	.0312
		n	$R^I - R^{II}$ mean	t	$P_{>t}$	$z, P_{>z}$
50	+	15	3.874	4.926	.0002	$z = 2.593$, $P_{>z} = .0095$

Table 3: Excess revenue for different treatments

The table compares excess revenue $R - R^*$ for different treatments. n is the number of independent observations. Mean excess revenue is shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). Data from the first 6 periods of each session is discarded.

Morgenstern utility must behave in a risk neutral way.

The idea we are proposing here, namely that bidders prefer simple linear bidding functions, is independent of the representation of payoffs as lottery tickets or as money and, thus, consistent with Cox, Smith, and Walker (1985). We have seen in section 2 that optimal simple linear bidding functions imply underbidding for small valuations. Our experiment has confirmed this underbidding for a large variety of scenarios. Simple linear bids seem to fit better with what we observe in the laboratory than risk-aversion.

References

Cox, J. C., B. Roberson, and V. L. Smith, 1982, Theory and behavior of single object auctions, in *Research in experimental economics*, ed. by V. L. Smith. JAI Press, Greenwich, Conn.

Cox, J. C., V. L. Smith, and J. M. Walker, 1983, Test of a Heterogeneous Bidder's Theory of First Price Auctions, *Economic Letters*, 12(3-4), 207–212.

——— , 1985, Experimental Development of Sealed-Bid Auction Theory: Calibrating Controls for Risk Aversion, *American Economic Review*, 75(2), 160–165.

Cox, J. C., V. L. Smith, and J. M. Walker, 1988, Theory and Individual Behavior of First-Price Auctions, *Journal of Risk and Uncertainty*, 1, 61–99.

Fischbacher, U., 1999, z-Tree. Toolbox for Readymade Economic Experiments, Working paper 21, IEW, University of Zurich.

Güth, W., R. Ivanova-Stenzel, and E. Wolfstetter, 2004, Bidding Behavior in Asymmetric Auctions: An Experimental Study, Discussion paper, Humboldt-University Berlin.

Ivanova-Stenzel, R., and D. Sonsino, 2004, Comparative Study of One Bid versus Two Bid Auctions, *Journal of Economic Behavior and Organization*, to appear.

Kagel, J. H., R. M. Harstad, and D. Levin, 1987, Information impact and allocation rules in auctions with affiliated private values: A laboratory study, *Econometrica*, 55, 1275–1304.

Kagel, J. H., and D. Levin, 1993, Independent private value auctions: Bidder behavior in first-, second- and third-price auctions with varying numbers of bidders., *Economic Journal*, 103, 868–879.

A List of independent observations

date	$\underline{\omega}$	$\underline{b}$	second-price	participants
20040518-17:41	-25	-125	0	8
20040518-17:41	-25	-125	0	8
20040519-10:37	-25	-125	0	8
20040519-10:37	-25	-125	0	8
20040518-10:19	0	-100	0	8
20040518-10:19	0	-100	0	6
20040518-12:15	0	-100	0	6
20040518-12:15	0	-100	0	6
20040519-17:39	0	-100	0	8
20040519-17:39	0	-100	0	6
20040518-15:55	0	0	0	8
20040518-15:55	0	0	0	8
20040519-12:33	0	0	0	8
20040519-12:33	0	0	0	8
20040602-14:11	25	-75	0	8
20040602-14:11	25	-75	0	8
20040602-16:03	25	-75	0	10
20040517-12:21	50	-50	0	8
20040517-12:21	50	-50	0	6
20040517-17:17	50	-50	0	8
20040517-17:17	50	-50	0	8
20031210-15:45	50	0	0	12
20031211-18:23	50	0	0	14
20031212-10:45	50	0	0	14
20040519-15:53	50	0	0	8
20040519-15:53	50	0	0	10
20041130-17:41	50	0	1	10
20041130-17:41	50	0	1	10
20041201-14:09	50	0	1	10
20041201-14:09	50	0	1	10
20041201-15:57	50	0	1	10
20041201-15:57	50	0	1	8

The parameter $\underline{b}$ is the smallest possible bid. In the $-$treatments $\underline{b} = \underline{\omega} - 100$ and in the $+$treatments $\underline{b} = \underline{\omega}$. The highest bid that participants could enter was always $\overline{\omega} + 100$.

B Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats. Being seated participants then obtained written instructions in german. These instructions very slightly depending on the treatment. In the following we give a translation of the instructions.

After answering control questions on the screen subjects entered the treatment described in the instructions. After completing the treatment they answered a short questionnaire on the screen and where then payed in cash. The experiment was done with the help of z-Tree (Fischbacher (1999)).

B.1 General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can — depending on your decision — gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. **During the experiment no communication is permitted.** Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to exclusion from the the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU (Experimental Currency Unit). Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and payed to you in **cash**. The conversion rate will be shown on your screen at the beginning of the experiment.

B.2 Information regarding the experiment

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct **12 rounds**. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are **two bidders**. In each round you will be allocated randomly to another participant for the auction. *Your co-bidder in the auction changes in every round.* The bidder with the highest bid has obtained the object. If bids are the same the object will be allocated randomly.

For the auctioned object you have a valuation in ECU. This valuation lies between x and $x + 50$ ECU[3] and is determined randomly in each round. The range from x to $x + 50$

[3]In the 0+ and 50+ treaments the valuation would be announced precisely: "This valuation lies between 0 and 50 ECU" in the 0+ treatment and "This valuation lies between 50 and 100 ECU" in the 50+ treatment. Whenever x is mentioned in the remaineder of the instruction the same comment applies: In the 0+ and 50+ treatments the valuation is always announced precisely.

will be shown to you at the beginning of the experiment on the screen and is the same in each round.[4] **From this range you will obtain in each round new and random valuations for the object.** The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from x to $x + 50$ from which also your valuations are drawn. All valuations between x and $x+50$ are equally probable. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. *You will not know the valuation of the other bidder.*

B.2.1 Experimental procedure

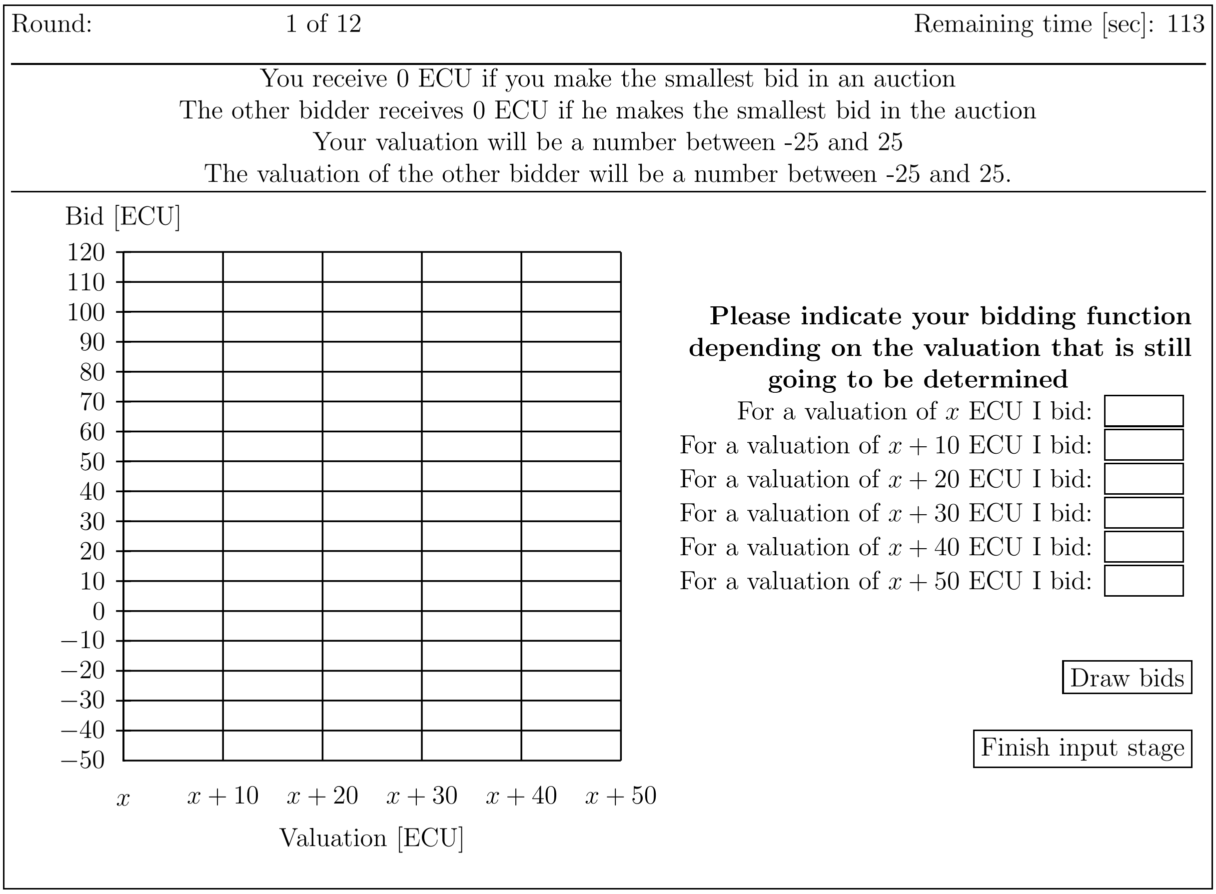
The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

1. Stage

In the first stage of the experiment you see the following screen:[5]

[4] This sentence was only part of the instructions in the 50-, 25-, 0-, and -25- treatments, though in all treatments the range was shown on the screen.

[5] In the instructions for the 50-, 25-, 0-, and -25-, the interval x to $x + 50$ was, as you see in the figure, described as x to $x + 50$. From the first round of the experiment on the current number where given. In the 0+ and 50+ treatments the interval was already shown exactly in the instructions and consistently also in the figures in the instructions.



At that stage **you do not know your own valuation for the object in this round.** On the right side of the screen you are asked to enter a bid for **six hypothetical valuations** that you might have for the object. These six hypothetical valuations are x, $x + 10$, $x + 20$, $x + 30$, $x + 40$, and $x + 50$ ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on "draw bids". In the graph the hypothetical valuation is shown on the horizontal axis, the bids are shown on the vertical axis. Your input in the table is shown as six points in the diagram. **Neighbouring points are connected with a line automatically.** These lines determine your bid for all valuations *between* the six points for those you have made an input. For the other bidder the screen in the first stage looks the same and there are as well bids for six hypothetical valuations. The other bidder can not see your input.

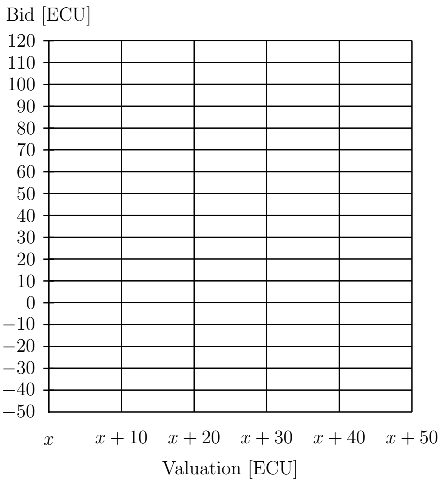
2. Stage

The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but **five auctions**. This is done as follows: **Five times a random valuation is determined** that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen:[6]

[6]In the instructions the following figure was shown. This figure does not show the bidding function in the graph and the specific bids, gains and losses that would be shown during the experiment.

Round: 1 of 12 Remaining time [sec]: 113

You receive 0 ECU if you make the smallest bid in an auction
The other bidder receives 0 ECU if he makes the smallest bid in the auction
Your valuation will be a number between -25 and 25
The valuation of the other bidder will be a number between -25 and 25.



Your income from all auctions in this round is . . . ECU

Auction 1:
Your randomly determined valuation is . . . ECU.
According to your entered bidding function you make a bid of . . . ECU.
You entered the larger bid.
Your income from this auction is . . . ECU.

Auction 2:
Your randomly determined valuation is . . . ECU.
According to your entered bidding function you make a bid of . . . ECU.
You entered the larger bid.
Your income from this auction is . . . ECU.

Auction 3:
Your randomly determined valuation is . . . ECU.
According to your entered bidding function you make a bid of . . . ECU.
You entered the smaller bid. The other bidder has made a bid of . . . ECU.
Your income from this auction is . . . ECU.

Auction 4:
Your randomly determined valuation is . . . ECU.
According to your entered bidding function you make a bid of . . . ECU.
You entered the smaller bid. The other bidder has made a bid of . . . ECU.
Your income from this auction is . . . ECU.

Auction 5:
Your randomly determined valuation is . . . ECU.
According to your entered bidding function you make a bid of . . . ECU.
You entered the larger bid.
Your income from this auction is . . . ECU.

Continue with the next round

For each of your five valuations the computer determines your bid according to the graph from stage 1. If a valuation is precisely at x, $x + 10$, $x + 20$, $x + 30$, $x + 40$, or $x + 50$ the computer takes the bid that you gave for this valuation. If a valuation is between these points your bid is determined according to the joining line. In the same way the bids of the other bidder are determined for his five valuations. Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.

Your income from the auction:

For each of the five auctions the following holds:

- The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.
- If the bidder with the higher bid has a negative valuation for the object, the ECU account is reduced by this amount.[7]

[7] This item is only shown in the 50-, 25-, 0- and 25- treatments. It is not shown in the 0+ and 50+ treatments.

Note that, in order to be able to use same instructions for all treatments we mention the possibility of negaive valuations in the 50-, 25-, 0-, and -25- treatments, even if in the 50- 25- and 0- treatments subjects learn later that their valuation is drawn from an interval that contains only positive numbers.

- If the bid of bidder with the higher is a negative number, the amount is added to his ECU account.[8]
- The bidder with the smaller bid obtains **no income** from this auction.

You total income in a round is **the sum of the ECU income from those auctions in this round where you have made the higher bid.**

This ends one round of the experiment and you see in the next round again the input screen from stage 1.

At the end of the experiment your total ECU income from all rounds will be converted into Euro and payed to you in cash together with your Show-Up Fee of 3.00 Euro.

Please raise your hand if you have questions.

[8] This item is only shown in the 50-, 25-, 0- and 25- treatments. It is not shown in the 0+ and 50+ treatments.

Nr.	Author	Title
04-70	Felix Freyland	Household Composition and Savings: An Empirical Analysis based on the German SOEP data
04-69	Felix Freyland	Household Composition and Savings: An Overview
04-68	Anette Reil-Held	Crowding out or crowding in? Public and private transfers in Germany.
04-67	Lothar Essig Anette Reil-Held	Chancen und Risiken der Riester-Rente
04-66	Alexander Ludwig Alexander Zimper	Rational Expectations and Ambiguity: A Comment on Abel (2002)
04-65	Axel Börsch-Supan Alexander Ludwig Joachim Winter	Aging, Pension Reform, and Capital Flows: A Multi-Country Simulation Model
04-64	Axel Börsch-Supan	From Traditional DB to Notional DC Systems&61482; Reframing PAYG contributions to ìnotional savingsî
04-63	Axel Börsch-Supan	Faire Abschläge in der gesetzlichen Rentenversicherung
04-62	Barbara Berkel Axel Börsch-Supan	Pension Reform in Germany: The Impact on Retirement Decisions
04-61	Axel Börsch-Supan Alexander Ludwig Anette Reil-Held	Projection methods and scenarios for public and private pension information
04-60	Joachim Schleich Karl-Martin Ehrhart Christian Hoppe Stefan Seifert	Banning banking in EU emissions trading?
04-59	Karl-Martin Ehrhart Christian Hoppe Joachim Schleich Stefan Seifert	The role of auctions and forward markets in the EU ETS: Counterbalancing the economic distortions of generous allocation and a ban on banking

Nr.	Author	Title
04-58	Stefan Seifert Karl-Martin Ehrhart	Design of the 3G Spectrum Auctions in the UK and in Germany: An Experimental Investigation
04-57	Karl-Martin Ehrhart Roy Gardner Jürgen von Hagen Claudia Keser*	Budget Processes: Theory and Experimental Evidence
04-56	Susanne Abele Karl-Martin Ehrhart	The Timing Effect in Public Good Games
04-55	Karl-Martin Ehrhart Christian Hoppe Joachim Schleich Stefan Seifert	Emissions Trading and the Optimal Timing of Production
04-54	Ralph W. Bailey Jürgen Eichberger David Kelsey	Ambiguity and Public Good Provision in Large Societies
04-53	Hendrik Hakenes Isabel Schnabel	Banks without Parachutes – Competitive Effects of Government Bail-out Policies
04-52	Hendrik Hakenes Martin Peitz	Selling Reputation When Going out of Business
04-51	Hendrik Hakenes Martin Peitz	Umbrella Branding and the Provision of Quality
04-50	Siegfried K. Berninghaus Bodo Vogt	Network Formation in Symmetric 2x2 Games
04-49	Ani Guerdjikova	Evolution of Wealth and Asset Prices in Markets with Case-Based Investors
04-48	Ani Guerdjikova	Preference for Diversification with Similarity Considerations
04-47	Simon Grant Jürgen Eichberger David Kelsey	CEU Preferences and Dynamic Consistency

Nr.	Author	Title
04-46	Ani Guerdjikova	A Note on Case-Based Optimization with a Non-Degenerate Similarity Function
04-45	Jürgen Eichberger Martin Summer	Bank Capital, Liquidity and Systemic Risk
04-44	Ani Guerdjikova	Asset Prices in an Overlapping Generations Model with Case-Based Decision Makers with Short Memory
04-43	Fabian Bornhorst Andrea Ichino Oliver Kirchkamp Karl H. Schlag Eyal Winter	How do People Play a Repeated Trust Game? Experimental Evidence
04-42	Martin Hellwig	Optimal Income Taxation, Public-Goods Provision and Public-Sector Pricing: A Contribution to the Foundations of Public Economics
04-41	Thomas Gschwend	Comparative Politics of Strategic Voting: A Hierarchy of Electoral Systems
04-40	Ron Johnston Thomas Gschwend Charles Pattie	On Estimates of Split-Ticket Voting: EI and EMax
04-39	Volker Stocké	Determinants and Consequences of Survey Respondentsí Social Desirability Beliefs about Racial Attitudes
04-38	Siegfried K. Berninghaus Marion Ott Bodo Vogt	Restricting the benefit flow from neighbors: Experiments on network formation
04-37	Christopher Koch	Behavioral Economics und die Unabhängigkeit des Wirtschaftsprüfers - Ein Forschungsüberblick
04-36	Christopher Koch	Behavioral Economics und das Entscheidungsverhalten des Wirtschaftsprüfers - Ein Forschungsüberblick

Nr.	Author	Title
04-35	Christina Reifschneider	Behavioral Law and Economics: Überlegungen zu den Konsequenzen moderner Rationalitätskonzepte für die Gestaltung informationellen Kapitalmarktrechts
04-34	Siegfried K. Berninghaus Karl-Martin Ehrhart Marion Ott Bodo Vogt	Searching for "Stars" - Recent Experimental Results on Network Formation -
04-33	Christopher Koch	Haftungserleichterungen bei der Offenlegung von Zukunftsinformationen in den USA
04-32	Oliver Kirchkamp J. Philipp Reiß	The overbidding-myth and the underbidding-bias in first-price auctions
04-31	Alexander Ludwig Alexander Zimper	Investment Behavior under Ambiguity: The Case of Pessimistic Decision Makers
04-30	Volker Stocké	Attitudes Toward Surveys, Attitude Accessibility and the Effect on Respondentsí Susceptibility to Nonresponse
04-29	Alexander Ludwig	Improving Tatonnement Methods for Solving Heterogeneous Agent Models
04-28	Marc Oliver Rieger Mei Wang	Cumulative Prospect Theory and the St.Petersburg Paradox
04-27	Michele Bernasconi Oliver Kirchkamp Paolo Paruolo	Do fiscal variables affect fiscal expectations? Experiments with real world and lab data
04-26	Daniel Schunk Cornelia Betsch	Explaining heterogeneity in utility functions by individual differences in preferred decision modes
04-25	Martin Weber Jens Wuestemann	Bedeutung des Börsenkurses im Rahmen der Unternehmensbewertung
04-24	Hannah Hörisch	Does foreign aid delay stabilization