Instructions for the exam

- You can start working once you have obtained the *task sheet*. You may receive the *answer sheet* later (when your ID is checked) — the answer sheet is only needed at the end of the exam. The answer sheet already contains your name and your matriculation number. Please make sure that you have obtained the right answer sheet (with your name).

  On the answer sheet you also find your PIN. Please write this number down and keep it in a safe place. You will need this number to get your exam results.

- At the end of the exam please *only* submit the *answer sheet*. You will keep the task sheet, your scratch paper and these instructions. If you mark your answers on the task sheet you can easily compare your answers with the solutions to the exam.

  The answer sheet will be collected in the same order in which the task sheet was distributed. Thus, everybody has approximately the same time to work on the exam. To make sure that nobody has an undue advantage late submissions do not count.

  Please remain *quietly at your place, until the last person has completed the exam*. Please respect the other students. Everybody should have the chance to complete their exam in a quiet environment.

- You **only** need **paper and pen** for this exam. If necessary, please discuss eventual needs for special support before the exam. Bags, calculators, mobile phones, books, etc. are left at the front of the room. Please make sure that you have completely switched off your mobile devices before you leave them.

- There are different version of the exam. Your version code is indicated at the top of the task sheet e.g. as follows:

  Version:
  ☒☐☒☐☐—☐☐☒☐☐

  Please copy your version code from the task sheet into the answer sheet.

- Please mark your answers with a dark pen.

answer x.x	a	b	c	d	e
	☐	☐	☐	☒	☐

  You only score points if you tick the correct answer. You obtain no points if you tick the wrong answer.

Please note that you obtain already some points by giving random answers. To pass you will need more points than what you can obtain with random answers.

The last five minutes:

- Please copy your answers into the answer sheet **only when you are sure about your answers.**

  If you, nevertheless, have to "clear" a field, please cross out the answer as in this example. Here only (b) is marked as correct.

answer x.x:	a	b	c	d	e
	☐	☒	☐	✸	☐

- Please do not forget to sign your answer sheet with indelible ink (e.g. a ballpoint pen). Also do not forget to write down your PIN in a safe place. You will need the PIN to get your exam results.

- We wish you good luck

Notation The following notation is used interchangeably to denote derivatives:

Leibniz's (1675) notation: $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ for the first and second derivative.

Lagrange's (1772) notation: f', f'' for the first and second derivative. This notation is shorter but assumes that we know that derivates are taken with respect to x.

We also write f_x and f_{xx} for the first and second derivative of f if we have to point out that derivatives are taken with respect to x.

Taking the first derivative of f' we obtain the second derivative f''.

Note that we write f and $f(x)$ interchangeably. The former is shorter, but it assumes that we know that f is actually a function of x.

Optimisation

- Minimum: $f'(x) = 0$ and $f''(x) > 0$

- Maximum: $f'(x) = 0$ and $f''(x) < 0$

Differentiation rules In the following a is a constant and u, v and f are functions of x.

$f =$	$f' =$	$f =$	$f' =$
a	0	x^n	nx^{n-1}
au	au'	e^x	e^x
$u + v$	$u' + v'$	$\ln x$	$1/x$
uv	$u'v + uv'$	$\sin x$	$\cos x$
$u(v)$	$u'(v)\, v'$	$\cos x$	$-\sin x$
$\dfrac{u}{v}$	$\dfrac{u'v - v'u}{v^2}$		