Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

MW24.3, 27. 7. 2022

You have 60 minutes to answer the following questions. Please explain all your answers. Good luck!

1. Two partners jointly work on a project and equally share the project's revenue. Each partner i chooses how much effort $e_i \in [0, 4]$ to invest into the joint project. Total revenue of the joint project is

$$\Pi(e_1, e_2) = 4(e_1 + e_2 + b e_1 e_2) \text{ with } b \in (0, 1/4]\,.$$

The cost of providing effort e_i for each partner i is

$$c_i(e_i) = e_i^2\,.$$

Assume that revenue $\Pi(e_1, e_2)$ is shared evenly. Then the payoff of each player i is

$$u_i(e_1, e_2) = \frac{1}{2}\Pi(e_1, e_2) - c_i(e_i)\,.$$

   a) Explain: What is the meaning of the parameter b? Does it make sense to assume that $b > 0$?

   b) Assume that partners can not write a contract on efforts. Which efforts e_1 and e_2 would players provide in the Nash equilibrium of this game?

   c) Assume that the two partners could observe their mutual efforts e_1 and e_2 and that they could write a contract on these efforts. Which efforts would the two partners specify to maximise the total profit. Will they invest more or less than without a contract? Explain your answer!

      Assume that the two partners share their revenue according to the Nash Bargaining Solution. The disagreement point is given by the Nash Equilibrium from 1b. How would the two partners share their gains from cooperation. Explain your answer!

   d) Assume now that the two partners share their revenue according to the Kalai Smorodinsky Solution. The disagreement point is still given by the Nash Equilibrium from 1b. How would the two partners share their gains from cooperation. Explain your answer!

2. Consider a bargaining problem for two players $\langle S, d\rangle$ where S is a set of pairs of utilities and d is the disagreement point. Consider the bargaining solution f^X given by

$$f^X(S, d) = \operatorname*{argmax}_{(d_1,d_2)\leq(s_1,s_2)\in S} (s_1 - d_1)^2 + (s_2 - d_2)$$

a) Under the usual assumptions on S and d, does a solution f^X always exist? Is it always unique? Explain your answers!

b) Which axioms that we discussed in the context of Nash's bargaining solution are fulfilled by f^X, and which are not? Explain your answers!

3. In our discussion of Rubinstein's model of bargaining with alternating offers we described preferences of players with the help of a function $v_i(x_i, t)$. This function describes the present value of an outcome x_i at time t for a player i.

$$v_i(x_i, t) = \begin{cases} y_i & \text{if } (y, 0) \sim_i (x, t) \\ 0 & \text{if } (y, 0) \succ_i (x, t) \text{ for all } y \in X \end{cases}$$

Consider the case where
$$v_i(x_i, 1) = x_i - \alpha x_i^2 \,.$$

a) Which values may α have, so that the axiom of "increasing loss to delay" is satisfied?

b) Which of the other axioms of Rubinstein's model of bargaining with alternating offers are not satisfied with $v_i(x_i, 1) = x_i - \alpha x_i^2$?

c) Assume $\alpha = \frac{1}{2}$ for both players.

   i. Which divisions are possible in the subgame perfect equilibrium? Explain your answer.

   ii. Which divisions are possible in the Nash equilibrium? Explain your answer!

d) Now consider a bargaining situation where player 2 has on outside option b. Each time after player 2 has rejected an offer of player 1, player 2 can decide to take the outside option. In this case player 2 obtains b and player 1 obtains 0. Still assume that $v_i(x_i, 1) = x_i - \alpha x_i^2$ with $\alpha = \frac{1}{2}$.

   What is now a subgame perfect equilibrium? Explain your answer!

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

MW24.3, 6. 8. 2021

You have 60 minutes to answer the following questions. Please explain all your answers. Good luck!

1. Consider the bargaining problem of splitting a pie of size 100 among two players. Players 1 and 2 have utilities $u_1(x_1) = x_1$ and $u_2(x_2) = x_2$. We call x_1 and x_2 the shares of player 1 and player 2, respectively. Disagreement allocations (the amounts of pie players obtain in case of disagreement) are zero for both players.
   a) What is the Nash bargaining solution for this problem if only divisions of the pie that give both players positive amounts are possible?
   b) Now (and for the following questions) assume $u_2(x_2) = 2x_2 - 1$. How is the pie divided according to the Nash bargainng solution?
   c) How is the pie divided if it is impossible to allocate more than 70 units to player 1?
   d) How is the pie divided if it is impossible to allocate more than 50 units to player 1?
   e) How is the pie divided if it is impossible to allocate more than 30 units to player 1?
   f) How does your answer to the last question change if the disagreement allocation is 10 units of pie for player 1 and 0 units of player 2?

2. Consider a bargaining problem with two players $\langle S, d\rangle$. S is a pair of utilities associated with possible outcomes of the bargaining process. d is the disagreement outcome. Consider the bargaining solution F given by the element $x \in S$ that maximises $x_1 + x_2 - (d_1 + d_2)$ under the restriction $x_1 - d_1 = 2(x_2 - d_2)$. Which of the standard axioms are satisfied by this solution, which are not satisfied. Explain!

3. A buyer wants to buy an item from a seller. The quality q of the item is known only to the seller. The buyer expects q to be uniformly distributed over $[0, 1]$. The seller's valuation of the item is q. The buyer's valuation of the item is $\frac{1}{2} + \frac{1}{2}q$. Buyers propose a price p. If the seller accepts, the buyer enjoys her valuation of the item minus the price. The seller obtains the price. If the seller rejects, the game ends and the seller still enjoys the value of the item.
   a) Draw the game tree.
   b) What is an appropriate equilibrium concept for this game?

c) Find all equilibria of the game. Explain your answer.

d) Consider a situation where the buyers expects that q can have only two values $q_1 < q_2$ with equal probability. Find all equilibria of this game.

4. Consider the following pair of strategies in a game of alternating offers with a constant discount factor δ. We call x_1 the share player 1 obtains in a proposal. We call x_2 the share of player 2. The initial state is A.

	A	B
1 proposes	$(x^*, 1 - x^*)$	$(0, 1)$
1 accepts	$x_1 \geq x^*$	$x_1 \geq 0$
2 proposes	$(x^*, 1 - x^*)$	$(0, 1)$
2 accepts	$x_2 \geq 1 - x^*$	$x_2 = 1$
transitions	go to B if a proposal was rejected	absorbing

a) For which values of x^* is this a Nash equilibrium? Explain!

b) For which values of x^* is this a subgame-perfect equilibrium? Explain!

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Bargaining Theory, 9. 8. 2013

You have 45 minutes to answer the following questions. Please explain your answers, and please write in a clear and readable way. You can only use pen and paper. Good luck!

1. Consider the bargaining problem of splitting a pie of size 100 with utilities $u_1(x_1) = x_1$ and $u_2(x_2) = x_2$ for players 1 and 2. The disagreement allocations (the amounts of pie players obtain in case of disagreement) are zero for both players.
   a) What is the Nash bargaining solution for this problem if any division of the pie that gives both players positive amounts is possible?
   b) Now (and for the following questions) $u_2(x_2) = 2x_2 - 1$. How is the pie divided according to the Nash bargainng solution?
   c) How is the pie divided if it is impossible to allocate more than 70 units to player 1?
   d) How is the pie divided if it is impossible to allocate more than 50 units to player 1?
   e) How is the pie divided if it is impossible to allocate more than 30 units to player 1?
   f) How does you answer to the last question change if the disagreement allocation is 10 units of pie for player 1 and 0 units of player 2?

2. In the lecture we studied the Rubinstein bargaining game with alternating offers. Both players i have a von Neumann-Morgenstern utility function $u(x_i) = x_i$. Both players have a discount factor of δ. Now consider the following modification:

   After a rejection the game does not necessarily continue with an offer of the other player. Instead players toss a fair coin. Each player has a chance of $\frac{1}{2}$ to be the next to make an offer.

   Consider the following combination of strategies where $x^* \equiv (x_1^*, x_2^*)$ and $y^* \equiv (y_1^*, y_2^*)$. x_1 and x_2 denote shares that are offered to players 1 and 2 respectively.

	$*$
1 proposes	x^*
1 accepts if	$x_1 \geq y_1^*$
2 proposes	y^*
2 accepts if	$x_2 \geq x_2^*$
	absorbing

a) What values of x^* and y^* are Nash equilibria? Explain!

b) What values of x^* and y^* are subgame-perfect equilibria? Explain!

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Bargaining Theory, 20. 2. 2009

You have 60 minutes to answer the following questions. Please explain your answers, and please write in a clear and readable way. You can only use pen and paper. Good luck!

1. Consider the bargaining problem of splitting a pie of size 1 with utility $u(x_1) = x_1$ for player 1 and $v(x_2) = 2x_2 - x_2^2$ for player 2, where x_1 and x_2 denote the share of the pie for player 1 and 2 respectively.
   a) Draw (approximately) the utility function for player 2 and explain why it is strictly increasing and concave.
   b) Draw (approximately) the utility possibility frontier.
   c) What is the Nash bargaining solution for this problem if the disagreement outcome (the utilities players obtain in case of disagreement) is $d_1 = d_2 = 0$?
   d) What is the Nash bargaining solution if the disagreement outcome is any d_1 and d_2?

2. Consider a modified version of the Rubinstein bargaining game with alternating offers. Both players i have a utility function $u(x_i) = x_i$. Both players have a discount factor of δ. In contrast to the original Rubinstein model players enjoy rejecting offers. Whenever player i says "no" to an offer she enjoys an extra utility a_i (this extra utility is also discounted with δ).

   E.g. if players agree in period 3 to (x_1, x_2), and player 1 has said "no" three times before (in period 0, 1, and 2) then her total utility is $a_1 + \delta a_1 + \delta^2 a_1 + \delta^3 x_1$.

   Assume that $\frac{a_1+a_2}{1-\delta} < 1$.
   a) Show that the sum of utilities is greater if players reach immediate agreement than if the say "no" to each other forever.
   b) Consider the following combination of strategies where $x^* \equiv (x_1^*, x_2^*)$ and $y^* \equiv (y_1^*, y_2^*)$. x_1 and x_2 denote offers made to players 1 and 2 respectively.

	$*$
1 proposes	x^*
1 accepts if	$x_1 \geq y_1^*$
2 proposes	y^*
2 accepts if	$x_2 \geq x_2^*$

      i. What values of x^* and y^* are Nash equilibria?

ii. What values of x^* and y^* are subgame-perfect equilibria?

c) How does, in the subgame-perfect equilibrium, x_1^* change if a_1 increases? Provide an intuition for your result?

d) What is the utility of player B in the subgame-perfect equilibrium?

Formula: $\sum_{i=0}^{\infty} \delta^i = \frac{1}{1-\delta}$

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Bargaining Theory, 20. 2. 2008

You have 60 minutes to answer the following questions. Please explain your answers, and please write in a clear and readable way. Good luck!

1. Consider a bargaining problem for two players $\langle S, d\rangle$ where S is a set of feasible pairs of utilites and d is the disagreement point. We consider the bargaining solution f'

$$f'(S,d) = \operatorname*{argmax}_{(d_1,d_2)\leq(s_1,s_2)\in S} (s_1 - d_1) + (s_2 - d_2)$$

   a) Which of the four Nash Axioms are satisfied by $f'()$ and which are not satisfied? Give a counterexample for each axiom that is not satiesfied and give a short proof for each axiom that is satisfied.

2. In games of alternating offers we have described preferences of a player i with the help of the following notation:

$$v_i(x_i, t) = \begin{cases} y_i & \text{if } (y,0) \sim_i (x,t) \\ 0 & \text{if } (y,0) \succ_i (x,t) \text{ for all } y \in X \end{cases}$$

   Consider the case where

$$v_1(x_1, 1) = \frac{1}{2}x_1$$

$$\text{and} \qquad v_2(x_2, 1) = \max\left(0, x_2 - \frac{1}{4}\right)$$

   a) Which axioms of the Rubinstein bargaining model are satiesfied by these preferences? Which are not satiesfied? Explain!
   b) Consider the standard game of alternating offers with two players. Given the above preferences, find a subgame perfect equilibrium of this game where player 1 makes a proposal in the first period and player 2 accepts.
   c) Find at least one Nash equilibrium of this game where player 1 makes a proposal in the first period and player 2 accepts and which leads to a different outcome than the subgame perfect equilibrium you found above.
   d) Is it possible to find a subgame perfect equilibrium where player 2 does not accept in the first period?

3. Now consider a game of alternating offers where preferences of players can be represented as

$$v_i(x_i, 1) = \alpha_i x_i \,.$$

It is common knowledge that $\alpha_1 = \frac{1}{2}$. However, the value of α_2 is only known to player 2. Player 1 only knows that α_2 can have two values. With probability $\frac{1}{2}$ we have $\alpha_2 = \frac{1}{2}$, and with probability $\frac{1}{2}$ we have $\alpha_2 = 0$.

   a) Can you find a pooling equilibrium of this game, i.e. an equilibrium where the play does not depend on the type of player 2? Explain!

   b) Can you find a separating equilibrium of this game, i.e. an equilibrium where the play depends on the type of player 2? Explain!

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Verhandlungstheorie, 3. 2. 2007

1. Betrachten Sie ein Verhandlungsproblem für zwei Spieler $\langle S, d\rangle$ in dem S und d die übliche Bedeutung haben (S ist eine Menge von Nutzenpaaren die mit möglichen Verhandlungsergebnissen assoziiert sind, und d ist der "disagreement point"). Sei die Verhandlungslösung f^{II} gegeben durch

$$f^{II}(S,d) = \underset{(d_1,d_2)\leq(s_1,s_2)\in S}{\operatorname{argmax}} (s_1-d_1)^2\cdot(s_2-d_2)$$

   a) Welche Axiome die wir im Zusammenhang mit der Nash Verhandlungslösung diskutiert haben, werden durch $f^{II}()$ erfüllt, und welche nicht? Geben Sie für jedes Axiom das nicht erfüllt ist ein Gegenbeispiel, und für jedes erfüllte Axiom einen kurzen Beweis an. Wenn der Beweis offensichtlich ist, erklären Sie warum!
   b) Geben Sie zwei Interpretationen für $f^{II}()$ an.

2. Nehmen Sie an, dass zwei Manager ein gemeinsames Projekt durchführen können. Wenn sie das tun, tätigt jeder Manager $i \in \{1,2\}$ eine Investition x_i. Die Entscheidung über x_i wird simultan gefällt, ohne dass die Manager wissen, was der jeweils andere Manager wählt.

   Danach realisiert sich ein gemeinsamer Gewinn $y(x_1,x_2) = x_1^\alpha \cdot x_2^\beta$ mit $\alpha \in [0,1]$, $\beta \in [0,1]$, $\alpha+\beta < 1$. Wie dieser gemeinsame Gewinn aufgeteilt wird, hängt von einem Parameter $\theta \in [0,1]$ ab: Manager 1 bekommt $\theta \cdot y(x_1,x_2)$ und Manager 2 bekommt $(1-\theta)\cdot y(x_1,x_2)$. Außerdem fallen Kosten an. Jede Einheit x_i verursacht genau eine Einheit Kosten.

   a) Nehmen Sie an, der Staat legt den Wert von θ fest. Welche Investitionen x_i wählen die Manager in Abhängigkeit von θ?
   b) Nehmen Sie nun an, die Manager legen vor der Entscheidung über ihre Investitionsniveaus den Wert von θ in einer Verhandlung fest. Falls die Verhandlungen über θ scheitern, erhalten beide einen Gewinn von 0. Das Ergebnis der Verhandlung sei die Nash Verhandlungslösung.
      i. Welchen Wert von θ werden sie wählen?
      ii. Welche Investitionsniveaus x_i werden sie wählen?
      iii. Wie groß ist der Gewinn der Manager?

c) Nun haben die Manager die Möglichkeit, in ihrer Verhandlung nicht nur den Wert von θ, sondern außerdem den Wert von x_1 und x_2 festzulegen. Das Ergebnis der Verhandlung sei wieder die Nash Verhandlungslösung.

   i. Welchen Wert von θ werden sie wählen?

   ii. Welche Investitionsniveaus x_i werden sie wählen?

   iii. Wie groß ist der Gewinn der Manager?

3. In der Diskussion des Rubinstein Verhandlungsmodells mit abwechselnden Vorschlägen haben wir die Präferenzen der Spieler durch eine Funktion $v_i(x_i, t)$ beschrieben, die den Gegenwartswert einer Aufteilung für Spieler i darstellt. Zur Erinnerung:

$$v_i(x_i, t) = \begin{cases} y_i & \text{if } (y, 0) \sim_i (x, t) \\ 0 & \text{if } (y, 0) \succ_i (x, t) \text{ for all } y \in X \end{cases}$$

Die Präferenzen der Spieler seien nun beschrieben durch

$$v_i(x_i, 1) = x_i - \alpha x_i^2 \,.$$

a) Welche Werte darf α nur annehmen, damit das Axiom von "increasing loss to delay" erfüllt ist?

b) Welche Axiome des Rubinstein Verhandlungsmodells sind mit dieser Funktion $v_i()$ nicht erfüllt?

c) Sei $\alpha = \frac{1}{2}$ für beide Spieler in einem Rubinsteinverhandlungsspiel mit zwei Spielern.

   i. Welche Aufteilungen sind im teilspielperfekten Gleichgewicht möglich? Geben Sie bitte eine Strategie an, die zu diesen Aufteilungen führt.

   ii. Welche Aufteilungen sind im Nash Gleichgewicht möglich? Geben Sie bitte eine Strategie an, die zu diesen Aufteilungen führt.

d) Nun verhandeln drei Spieler wie in dem Modell mit drei Spielern das in der Vorlesung behandelt wurde. Wieder sei $\alpha = \frac{1}{2}$. Was ist nun ein teilspielperfektes Gleichgewicht?

e) Betrachten Sie ein Rubinstein Verhandlungsmodell mit drei Spielern und konstantem Diskontfaktor δ. Reihum macht einer der drei Spieler einen Vorschlag wie in Kuchen aufzuteilen ist. Wenn mindestens *einer* der beiden anderen Spieler zustimmt, wird der Kuchen so aufgeteilt, und das Spiel endet, ansonsten vergeht eine Periode und in der nächsten Periode macht der nächste Spieler einen Vorschlag.

   i. Finden Sie ein teilspielperfektes Gleichgewicht dieses Spiels.

   ii. Finden Sie alle teilspielperfekten Gleichgewichte dieses Spiels.

f) Wie ist es, wenn jeweils der nächste Spieler zustimmen muss? Das heißt, nach einem Vorschlag von Spieler 1 muss Spieler 2 zustimmen, nach einem Vorschlag von Spieler 2 muss Spieler 3 zustimmen, und nach einem Vorschlag von Spieler 3 muss Spieler 1 zustimmen.

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Exam EC5501 Candlemas Semester 2005

A-formal Consider the Rubinstein bargaining game with alternating offers that we studied in the lecture. Remember that in this game player i would obtain a payoff of $u_i = \delta^n x$ if agreement was reached after n periods and this player gets a share of size x.

a) What if players, instead, pay a fixed waiting cost in each period. I.e. the payoff of player i is $u_i = x - n \cdot c_i$ where c_i is the individual waiting cost (which is small compared to the size of the pie). What can you say if players have different waiting cost $c_1 < c_2$?

b) What if players have the same waiting cost $c_1 = c_2$?

c) What if players have different waiting cost $c_1 > c_2$?

d) Do you think that real players would play the game like this? Please explain your answer.

A-formal Consider the following game which also leads to a division of a pie. Two players simultaneously state a share of the pie x_i and x_j that they want. If their claims are compatible (i.e. $x_i + x_j \leq 1$) they will get what they have claimed (player 1 gets x_1 and player 2 gets x_2).

a) What is the appropriate equilibrium concept? What are the equilibria of this game?

b) Can one use iterated elimination of dominated strategies in this game to find equilibria or to narrow down the set of equilibria? Explain all steps your answer clearly.

c) If you have found several equilibria in the first part of this question, what equilibrium do you expect will be played. Justify your answer.

d) What might influence this outcome.

B-essay format In the lecture we discussed bargaining situations in which bargaining power is very unevenly distributed. We have seen that behaviour in these situations does not always coincide with game theoretic predictions.

a) Discuss different explanations for this kind of behaviour. How can different motives be related to different participants in the bargaining process?

b) How can one distinguish between these explanations?

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Bargaining Theory, 14. 10. 2004

Explain all your answers in a clear, concise and legible way. Make clear which of your answers belongs to which question. Make also clear what is an answer and what is only an intermediate result. If you can not find an answer for some of the questions in the given time, explain clearly and briefly how you would proceed if you had more time. If you come to the conclusion that in a given case an equilibrium does not exist, explain why it does not exist. Write clearly and legibly!

1. Consider a two player bargaining problem $\langle S, d\rangle$ where S and d have the usual interpretation (S is a set of pairs of utilities associated with possible outcomes of the bargaining process and d is the disagreement point). Be F^E the element $x \in S$ that maximises x_1, i.e. the outcome preferred by player 1.
   a) Under the usual assumptions on S and d, does a solution F^E always exist? Is it always unique? If not, give a counterexample, if yes, give a brief proof. If the proof is obvious, explain why!
   b) Which axioms that we discussed in the context of Nash's bargaining solution are fulfilled by F^E, and which are not? Give a counterexample for each axiom that is not fulfilled and give a brief proof for each axiom that is fulfilled. If the proof is obvious, explain why!
2. Consider the following pair of strategies in a game of alternating offers with a constant discount factor δ. The share of player 1 is called x_1, the share of player 2 is called x_2.

	A	B
1 proposes	$(x^*, 1-x^*)$	$(1,0)$
1 accepts	$x_1 \geq \delta x^*$	$x_1 = 1$
2 proposes	$(\delta x^*, 1-\delta x^*)$	$(1,0)$
2 accepts	$x_2 \geq \delta - \delta^2 x^*$	$x_2 \geq 0$
transitions	go to B if a proposal was rejected	

a) For which values of x^* is this a Nash equilibrium?

b) For which values of x^* is this a subgame-perfect equilibrium?

3. Consider the model of a steady state market with decentralised trade. Sellers and buyers always choose the Nash solution when bargaining. Be the number of sellers S only slightly larger than the number of buyers B. In contrast to the model discussed in the lecture, let us assume that traders which are not matched in a given period have priority in the next period, i.e. they will be matched before any other, newly arrived traders are matched. If you wish, you can imagine them being in a queue where new traders always enter the end of the queue.

   If S is only slightly larger than B, what is a price p in the market?

4. Consider a standard bargaining game with alternating offers, but now with four players. In the first round player 1 makes a proposal how a cake of a fixed size is distributed among the four. Then the other three decide simultaneously whether to accept or reject. Only if the three unanimously accept the proposal the game ends and the proposal is implemnted. Otherwise the cake shrinks by a factor of δ and player 2 makes a proposal, the other three decide simultaneously..., then player 3 makes a proposal, the other three decide simultaneously..., then player 4 makes a proposal, the other three decide simultaneously,...and then the game continues again with player 1 and goes on as described above.

   Illustrate your answers to the following questions with the help of an example is possible.

   a) Is there a Nash equilibrium where agreement is reached in the first round that the cake is divided evenly?

   b) Is there a subgame perfect equilibrium where agreement is reached in the first round that that the cake is divided evenly?

   c) Is there a subgame perfect equilibrium where agreement is reached in the first round that player 2 gets all the cake?

   d) Is there a subgame perfect equilibrium where agreement is reached only in the fifth round?

   e) Does the solution of the game change if the three remaining players do not have to accept unanimously but a majority is sufficient?

   f) What is if only a single player must accept for the game to end and the proposal to be implemented?

Wirtschaftswissenschaftliche Fakultät
Prof. Dr. Oliver Kirchkamp
Lehrstuhl für Empirische und
Experimentelle Wirtschaftsforschung

Bargaining Theory, 30. 7. 2004

Explain all your answers in a clear, concise and legible way. Make clear which of your answers belongs to which question. Make also clear what is an answer and what is only an intermediate result. If you can not find an answer for some of the questions in the given time, explain clearly and briefly how you would proceed if you had more time. If you come to the conclusion that in a given case an equilibrium does not exist, explain why it does not exist. Write clearly and legibly!

1. Consider a two player bargaining problem $\langle S, d\rangle$ where S and d have the usual interpretation (S is a set of pairs of utilities associated with possible outcomes of the bargaining process and d is the disagreement point). Be F^E the so called 'egalitarian solution', i.e. the element $x \in S$ that maximises $x_i - d_i$ under the restriction $x_1 - d_1 = x_2 - d_2$.
   a) Under the usual assumptions on S and d, does a solution F^E always exist? Is it always unique? If not, give a counterexample, if yes, give a brief proof. If the proof is obvious, explain why!
   b) Which axioms that we discussed in the context of Nash's bargaining solution are fulfilled by F^E, and which are not? Give a counterexample for each axiom that is not fulfilled and give a brief proof for each axiom that is fulfilled. If the proof is obvious, explain why!
2. Be $\langle S, d\rangle$ a two player bargaining problem, f^N the Nash bargaining solution and $u(x)$ a utility transformation for player 1. To describe risk aversion we assume that u is monotonically increasing and concave. The bargaining problem $\langle S', d'\rangle$ is obtained by tranforming utilities of player 1 in $\langle S, d\rangle$ according to u, i.e. $S' = \{(u(x_1), x_2) | (x_1, x_2) \in S\}$ and $d' = (u(d_1), d_2)$. Compare the transformation of the Nash bargaining solution of the original problem $\langle S, d\rangle$ with the Nash bargaining solution of the transformed problem $\langle S', d'\rangle$. Explain!
3. Consider the following pair of strategies in a game of alternating offers with a constant discount factor δ. The share of player 1 is called x_1, the share of player 2 is called x_2.

	A	B
1 proposes	$(x^*, 1-x^*)$	$(1,0)$
1 accepts	$x_1 \geq x^*$	$x_1 = 1$
2 proposes	$(x^*, 1-x^*)$	$(1,0)$
2 accepts	$x_2 \geq 1-x^*$	$x_2 \geq 0$
transitions	go to B if a proposal was rejected	

a) For which values of x^* is this a Nash equilibrium?

b) For which values of x^* is this a subgame-perfect equilibrium?

4. Consider a two-player bargaining game with alternating offers over a pie that has value 1 and both bargainers have a fixed cost of delay. If no agreement is reached player 1 faces a cost c_1 and player 2 faces a cost c_2 so that the player i's payoff if agreement is reached in period t and if player i's share of the pie is x_i is given as $u_i(x_i, t) = x - t \cdot c_i$.

   a) What can you say about the subgame perfect equilibria of this game?

   b) In case 4a the accumulated waiting cost could be larger than the value of the pie if the agreement was reached very late. Assume now that the player's accumulated waiting cost is limited and can not be larger than the value of the pie so that the player i's payoff in period t, if the player's share of the pie is x_i, is given as $u_i(x_i, t) = \max\{0, x - t \cdot c_i\}$. What can you say about the subgame perfect equilibria of this game?

5. Two players interact for T periods. In each period one of them makes an offer s how to divide the joint profit they could make in each period. In the first period player 1 makes an offer, if player 2 does not accept, player 2 makes an offer in period 2, if player 1 does not accept, player 1 makes an offer in period 3 etc.. When players do not accept they make no profit in this period. As soon as one player accepts, they stop making offers and start making a profit of 1 in each remaining period, and player 1 gets a share of s and player 2 gets a share if $1-s$ in each of the remaining periods until period T where the game ends.

   a) What is the subgame-perfect equibrium of this game if $T = 1$?

   b) What is the subgame-perfect equibrium of this game if $T = 2$?

   c) What is the subgame-perfect equibrium of this game in the limit if $T \to \infty$?

d) What can you say about Nash-equilibria in this game if $T = 1$.

e) What can you say about Nash-equilibria in this game in the limit if $T \to \infty$.

f) Now assume that players discount their payoff with a factor δ which is common for both players. What is now a subgame perfect equilibrium for a given T? What, if $T \to \infty$?