Distributions in R:

type	distribution (F)	quantile (Q)
normal distribution	`pnorm`	`qnorm`
t-distribution	`pt`	`qt`
χ^2-distribution	`pchisq`	`qchisq`
F-distribution	`pf`	`qf`

Poisson distribution: $P_\lambda(X=k) = \lambda^k \cdot e^{-\lambda}/k!$; $E[X]=\lambda$; $\mathrm{var}(X)=\lambda$

Exponential distribution: $f_\lambda(X) = \begin{cases} \lambda e^{-\lambda X} & X \geq 0 \\ 0 & \text{otherwise} \end{cases}$;

$F_\lambda(X) = \begin{cases} 1-e^{-\lambda X} & X \geq 0 \\ 0 & \text{otherwise} \end{cases}$;

$E[X] = 1/\lambda$; $\mathrm{var}(X) = 1/\lambda^2$

Some integrals: $\int x\,dx = \frac{1}{2}x^2 + C$; $\int x^n\,dx = x^{n+1}/(n+1) + C$; $\int \frac{1}{x}\,dx = \ln x + C$; $\int a^x\,dx = a^x/\ln a + C$

Derivative of the Log-Likelihood function:

$$\frac{d}{d\theta}\ln L(x_1,\ldots,x_n|\theta) = \frac{f'(x_1|\theta)}{f(x_1|\theta)} + \cdots + \frac{f'(x_n|\theta)}{f(x_n|\theta)}$$

Expected value: $E(c\cdot X) = c\cdot E(X)$; $E(X+Y) = E(X)+E(Y)$

Variance: $\mathrm{var}(c\cdot X) = c^2\cdot\mathrm{var}(X)$; $\mathrm{var}(X+Y) = \mathrm{var}(X)+\mathrm{var}(Y)+2\cdot\mathrm{cov}(X,Y)$

Variance of $\bar{X}$: $\mathrm{var}(\bar{X}) = \sigma_X^2/n$

Standard deviation of $\bar{X}$: $\sigma_{\bar{X}} = \sigma_X/\sqrt{n}$

Estimator for expected value: $\hat{\mu}_X = \bar{X} = \frac{1}{n}\sum_i X_i$

Estimator for variance: $\hat{\sigma}_X^2 = \frac{1}{n-1}\sum_{i=1}^n \left(X_i - \bar{X}\right)^2$

Estimator for standard deviation of X:

$\hat{\sigma}_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^n \left(X_i-\bar{X}\right)^2}$

Estimator for $\sigma_{\bar{X}}$: $\hat{\sigma}_{\bar{X}} = \hat{\sigma}_X/\sqrt{n}$

Bias: $\mathrm{Bias}(\hat{\theta},\theta) = E(\hat{\theta}) - \theta$

Confidence interval for the mean:

$\left[\bar{X} + \sigma_{\bar{X}}\cdot Q\left(\frac{\alpha}{2}\right); \bar{X} - \sigma_{\bar{X}}\cdot Q\left(\frac{\alpha}{2}\right)\right]$

Type I and II error:

		actual condition	
		H_0 false	H_0 true
test result	reject H_0 (positive)	$1-\beta$, Power sensitivity	α, significance type I err.
	do not reject H_0 (negative)	β type II err.	$1-\alpha$ specificity

Comparing means (independent samples)

$$\frac{\bar{X}_A - \bar{X}_B}{\sqrt{\frac{\hat{\sigma}_A^2}{n_A}+\frac{\hat{\sigma}_B^2}{n_B}}} \sim t_{n_A+n_B-2}$$

Comparing means (paired samples)

$g = \frac{\bar{\Delta}}{\hat{\sigma}_{\bar{\Delta}}} \sim t_{n-1}$ mit $\Delta_i = X_i - Y_i$ und

$\hat{\sigma}_{\bar{\Delta}} = \frac{1}{\sqrt{n}}\sqrt{\frac{\sum_{i=1}^n(\Delta_i-\bar{\Delta})^2}{n-1}}$

χ^2-independence test $e_{ij} = \frac{\sum_{j=1}^k X_{ij}\cdot\sum_{i=1}^n X_{ij}}{\sum_{i=1}^n\sum_{j=1}^k X_{ij}}$

$g = \sum_{i=1}^n\sum_{j=1}^k \frac{\left(X_{ij}-e_{ij}\right)^2}{e_{ij}} \sim \chi^2_{(n-1)\cdot(k-1)}$

χ^2-goodness-of-fit test: $g = \sum_{i=1}^k \frac{(X(a_i)-n\cdot P(a_i))^2}{n\cdot P(a_i)} \sim \chi^2_{k-1}$

Testing means: $g = \frac{\bar{X}-\mu_0}{\hat{\sigma}_{\bar{X}}}$. If $X \sim N: g \sim t_{n-1}$ or $g^2 \sim F_{1,n-1}$.

If $n\to\infty: g \sim N(0,1)$ or $g^2 \sim F_{1,\infty}$.

AIC $= -2\cdot L + 2\cdot k$ (L is the Likelihood of the model, k is the number of parameters).

Logistic function: $\mathcal{L}(x) = 1/(1+e^{-x})$

$\mathrm{odds}(x) = \mathcal{L}(x)/(1-\mathcal{L}(x)) = e^x$

Precision: $\tau = 1/\sigma^2$

Conjugate priors:

$X \sim N(\mu,\tau)$, $\mu \sim N(\mu_0,\tau_0)$:

$$\mu_{\text{post}} = \frac{\tau_0\mu_0 + n\tau\bar{x}}{\tau_0+n\tau}$$
$$\tau_{\text{post}} = \tau_0 + n\tau$$

$X \sim N(\mu,\tau)$, $\tau \sim \Gamma(\alpha_0,\beta_0)$:

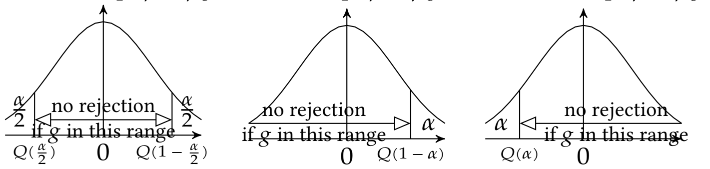
$$\text{shape} \quad \alpha_{\text{post}} = \alpha_0 + \frac{n}{2}$$
$$\text{rate} \quad \beta_{\text{post}} = \beta_0 + \frac{\sum_{i=1}^n(x_i-\mu)^2}{2}$$

$X \sim \mathrm{bern}(p)$, $p \sim \mathrm{Beta}(\alpha_0,\beta_0)$

$$\alpha_{\text{post}} = \alpha_0 + \sum x_i$$
$$\beta_{\text{post}} = \beta_0 + n - \sum x_i$$

Test of significance: test statistic: $g = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

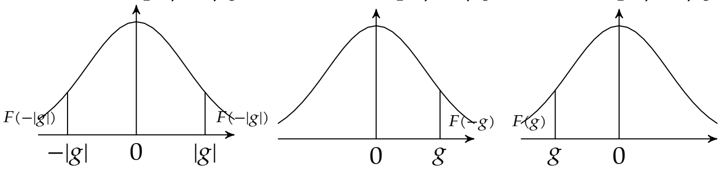
two sided ($H_1: \mu \neq \mu_0$) one sided ($H_1: \mu > \mu_0$) one sided ($H_1: \mu < \mu_0$)



H_0 is rejected if g is outside the range.

p-value: test statistic: $g = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

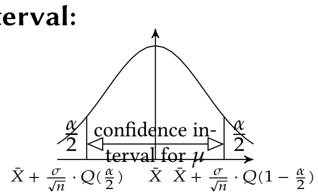
two sided ($H_1: \mu \neq \mu_0$) one sided ($H_1: \mu > \mu_0$) one sided ($H_1: \mu < \mu_0$)



$p = 2\cdot F(-|g|)$ $\quad$ $p = F(-g)$ $\quad$ $p = F(g)$

H_0 is rejected if $p < \alpha$.

Confidence interval:



$H_0: \mu = \mu_0$ is rejected if μ_0 is outside the confidence interval for μ.

	0.9	0.95	0.975	0.99	0.995	0.9975	0.999
qnorm(x)	1.28	1.64	1.96	2.33	2.58	2.81	3.09
qt(x,1)	3.08	6.31	12.71	31.82	63.66	127.32	318.31
qt(x,2)	1.89	2.92	4.30	6.96	9.92	14.09	22.33
qt(x,3)	1.64	2.35	3.18	4.54	5.84	7.45	10.21
qt(x,4)	1.53	2.13	2.78	3.75	4.60	5.60	7.17
qt(x,5)	1.48	2.02	2.57	3.36	4.03	4.77	5.89
qt(x,6)	1.44	1.94	2.45	3.14	3.71	4.32	5.21
qt(x,7)	1.41	1.89	2.36	3.00	3.50	4.03	4.79
qt(x,8)	1.40	1.86	2.31	2.90	3.36	3.83	4.50
qt(x,9)	1.38	1.83	2.26	2.82	3.25	3.69	4.30
qt(x,10)	1.37	1.81	2.23	2.76	3.17	3.58	4.14