Exams Auction Theory

Contents

1 Auction theory, 27. 7. 2004

The exam was set at Mannheim University. Participants had 90 minutes to answer all questions. The exam sheet also contained the formulas from appendix A.

Explain all your answers in a clear, concise and readable way. Make clear which of your answers belongs to which question. Make also clear what is an answer and what is only an intermediate result. If you can not find an answer for some of the questions in the given time, explain clearly and briefly how you would proceed if you had more time. If you come to the conclusion that in a given case an equilibrium bidding function does not exist, explain why it does not exist.

1. Consider an auction with an entry fee. Assume that the seller has a valuation of zero for the object.

   (a) Can it be good for the seller to charge an entry fee? Give a brief intuition? Which effects does one have to consider?

   (b) Is the resulting allocation in a standard auction with private values with an entry fee always efficient? Explain!

   (c) What other problem do buyers and sellers face in such a situation?

2. Two bidders have private valuations X_1 and X_2.

   (a) Both valuations are distributed independently and uniformly over $[0, 1]$. What is an equilibrium bidding function in a first price auction?

   (b) What is in this situation an equilibrium bidding function in a second price auction?

   (c) Now the valuation of bidder 2 is distributed uniformly over $[0, 2]$. What is now an equilibrium bidding function in a second price auction?

   (d) What is an equilibrium bidding function in a first price auction in this case?

3. Consider an auction with two bidders and interdependent valuations. Bidder 1 obtains a private signal x_1 and Bidder 2 obtains a private signal x_2. The two signals are independently distributed. The valuation for bidder 1 is

$$v_1 = (1 - \alpha)x_1 + \alpha x_2$$

   and the valuation for bidder 2 is

$$v_2 = (1 - \alpha)x_2 + \alpha x_1 \,.$$

   (a) Assume that bidders follow a symmetric equilibrium bidding strategy. For what values of α can no efficient allocation be obtained? How does the result depend on the distribution of each signal. Explain!

   (b) Assume that signals are distributed independently and uniformly over $[0, 1]$. What is an equilibrium bidding strategy in a second price auction?

   (c) What is an equilibrium bidding strategy in an English auction? (model the English auction as we did in the lecture)

4. Consider the following variant of an all-pay auction. There are two bidders, 1 and 2, with private valuations X_1 and X_2 for an object. They make bids b_1 and b_2 respectively. The bidder with the highest bid obtains the object. Regardless of who is the winner of the auction bidder 1 always pays $\frac{1}{2}b_1$ and bidder 2 always pays b_2.

(a) Can one use the revenue-equivalence principle to determine an equilibrium bidding function?

(b) If not, why not?

(c) If so, how could one proceed.

(d) Find an equilibrium.

5. Consider the following second-price all-pay auction. There are two bidders, 1 and 2 who make bids b_1 and b_2 respectively. The bidder with the highest bid obtains a fixed price P. Regardless of who is the winner of the auction, both bidders pay a cost which is characterised by the smaller bid $\underline{b} = \min\{b_1, b_2\}$ and individual parameters for each bidder, θ_1 and θ_2. Bidder 1 pays $c_1 = \underline{b} \cdot \theta_1$, bidder 2 pays $c_2 = \underline{b} \cdot \theta_2$.

   (a) Assume that both bidders know that $\theta_1 = \theta_2$. What can you say about equilibrium bidding strategies?

   (b) Assume that $\theta_1 = \frac{1}{2}\theta_2$. What can you say about equilibrium bidding strategies?

   (c) Assume that $\theta_1 = 1$ and θ_2 is uniformly distributed over $[\frac{1}{2}, \frac{3}{2}]$. What can you now say about equilibrium bidding strategies?

   (d) Assume that both θ_1 and θ_2 are uniformly distributed over $[\frac{1}{2}, \frac{3}{2}]$. What can you now say about equilibrium bidding strategies?

2 Auction theory, 14. 10. 2004

The exam was set at Mannheim University. Participants had 90 minutes to answer all questions. The exam sheet also contained the formulas from appendix A.

Explain all your answers in a clear, concise and readable way. Make clear which of your answers belongs to which question. Make also clear what is an answer and what is only an intermediate result. If you can not find an answer for some of the questions in the given time, explain clearly and briefly how you would proceed if you had more time. If you come to the conclusion that in a given case an equilibrium bidding function does not exist, explain why it does not exist.

Consider an auction with two bidders, bidder 1 and bidder 2. Both bidders submit simultaneously their bids b_1 and b_2. Bidder 1 wins if $b_1 >$

b_2. Bidder 2 wins if $b_2 > b_1$. If $b_1 = b_2$ the winner is determined by tossing a coin.

1. Consider an All-pay auction. Bidder 1 pays only one half of his bid $b_1/2$. Bidder 2 pays the complete bid b_2.

   (a) Valations X_1 and X_2 for both bidders are distributed according to the function $F(x)$. Can one use the revenue equivalence priciple to determine equilibrium bidding functions?

   (b) Determine the equilibrium bidding functions under the assumption that valuations X_1 and X_2 are distributed uniformly over $[0, 1]$.

   (c) Now bidders have the same valuation X^*. What are they doing in equilibrium?

2. Now consider the following variant of a first price auction. Bidder 1 pays, if he wins, one half of his bid $b_1/2$. Bidder 2 pays, if he wins, his bid b_2.

   (a) Valations X_1 and X_2 for both bidders are distributed according to the function $F(x)$. Can one use the revenue equivalence priciple to determine equilibrium bidding functions?

   (b) Determine the equilibrium bidding functions under the assumption that valuations X_1 and X_2 are distributed uniformly over $[0, 1]$.

   (c) Now bidders have the same valuation X^*. What are they doing in equilibrium?

3. Now consider the following variant of a second price auction. Bidder 1 pays, if he wins, $b_2/2$, bidder 2 pays, if he wins, b_1.

   (a) The situation is similar to a second price auction. Is it a dominant strategy to bid his true valuation X? If not, is there any other dominant strategy?

   (b) Valations X_1 and X_2 for both bidders are distributed according to the function $F(x)$. Can one use the revenue equivalence priciple to determine equilibrium bidding functions?

(c) Determine the equilibrium bidding functions under the assumption that valuations X_1 and X_2 are distributed uniformly over $[0, 1]$.

(d) Now bidders have the same valuation X^*. What are they doing in equilibrium?

4. Which of the above three auction types would you choose — given the above assumptions — if you are the seller?

   (a) Assume first a risk-neutral seller.

   (b) Now assume a risk-averse seller. How can you model risk-aversion? Can you find a specific functional form of a utility function to determine the utility maximising auction under risk aversion. Can you make a more general statement without assuming a specific function form?

5. Consider the following situation. Two bidders have valuations X_1 and X_2 for an object. Valuations are distributed according to $F(x)$. Bidders participate sequentially in the auction. First the auction determines a buy-now price p. This price is known to both bidders. Then bidder 1 submits his bid b_1. Bidder 2 does not observe b_1. If $b_1 \geqslant p$ then bidder 1 obtains the object and the auction ends. Otherwise bidder 2 can submit his bid b_2 and the bidder with the highest bid wins. Assume first the the winner pays the second highest bid.

   (a) What is an equilibrium strategy of bidder 2

   (b) What is an equilibrium strategy of bidder 1

   (c) What buy-now price is chosen by the revenue maximising auctioneer.

6. Now assume that the winner pays the own bid.

   (a) What is an equilibrium strategy of bidder 2

   (b) What is an equilibrium strategy of bidder 1

   (c) What buy-now price is chosen by the revenue maximising auctioneer.

3 Auction theory, 13. 5. 2005

The exam was set at St. Andrews University as a part of EC 4203. Participants had 30 minutes to answer all questions. The exam sheet also contained the formulas from appendix A.

1. An important theorem in auction theory is the revenue equivalence theorem. On which assumptions is the theorem based and what does it say? (25%)

2. Consider the situation of two bidders who have private valuations x_i for one good. These x_i are distributed independently and uniformly over $[0, 1]$. In the lecture we have found that for second-price auctions the weakly dominant bidding strategy for each bidder is to bid simply the own valuation, i.e. $\beta^{II}(x) = x$. Let us call the valuation of the other bidder y, then the expected payment of a bidder with valuation x is in the second-price auction $m^{II}(x) = \int_0^x y\, dy = x^2/2$. Now consider a different auction institution, not a second-price auction, but a first-price all-pay auction, i.e. all bidders pay their bid, but only the highest bidder wins the object.

   Use the revenue equivalence theorem to derive from the equilibrium of the second-price auction the symmetric equilibrium bidding function $\beta^{AP}(x)$ for first-price all-pay auction. (25%)

3. Give an intuition for the curvature of the bidding function in the first-price all-pay auction. (25%)

4. An all-pay auction looks like a strange institution. Where do we find all-pay auctions in real life? (25%)

4 Auktionstheorie, 5. 2. 2007

The exam was set at the University of Jena. Participants had 60 minutes to answer all questions. The exam sheet also contained the formulas from appendix A.

1. In einer Auktion mit drei Bietern und privaten Werten sind die Bewertungen der Bieter unabhängig voneinander und gleichverteilt über das Intervall $[0, 3]$.

(a) Was ist die symmetrische Bietfunktion in einer Erstpreisauktion?

(b) Was ist die symmetrische Bietfuntkion in einer Zweitpreisauktion?

(c) Was ist die symmetrische Bietfunktion in einer All-Pay Auktion?

2. In einer Auktion mit drei Bietern und privaten Werten sind die Bewertungen der Bieter unabhängig voneinander und verteilt entsprechend $F(x) = x^2$.

   (a) Was ist die symmetrische Bietfunktion in einer Erstpreisauktion?

   (b) Was ist die symmetrische Bietfuntkion in einer Zweitpreisauktion?

   (c) Was ist die symmetrische Bietfunktion in einer All-Pay Auktion?

3. In der Vorlesung haben wir die symmetrische Bietfunktion in der Erstpreisauktion im Fall zweier Bieter und exponentialverteilten Bewertungen (d.h. $F(x) = 1 - e^{-\lambda x}$) hergeleitet. Das Ergebnis war $\beta^I(x) = \frac{1}{\lambda} - \frac{x}{e^{\lambda x}-1}$.

   Verwenden Sie dieses Ergebnis um die symmetrische Bietfunktion für die All-Pay Auktion herzuleiten.

4. Betrachten Sie eine Auktion mit zwei Bietern und Werten die unabhängig und gleichverteilt sind über das Intervall $[0, 1]$.

   (a) Bieter 1 weiß, dass Bieter 2 immer der Bietfunktion $\beta_2 = \frac{x}{2}$ folgt. Was ist eine beste Antwort von Bieter 1 in einer Erstpreisauktion? Was ist die beste Antwort von Bieter 1 in einer Zweitpreisauktion?

   (b) Bieter 1 weiß, dass Bieter 2 immer der Bietfunktion $\beta_2 = \frac{1}{2} - \frac{x}{2}$ folgt.

   Was ist eine beste Antwort von Bieter 1 in einer Erstpreisauktion? Was ist die beste Antwort von Bieter 1 in einer Zweitpreisauktion?

   (c) Bieter 1 weiß, dass Bieter 2 immer der Bietfunktion $\beta_2 = x - \frac{1}{2}$ folgt.

   Was ist eine beste Antwort von Bieter 1 in einer Erstpreisauktion? Was ist die beste Antwort von Bieter 1 in einer Zweitpreisauktion?

(d) Bieter 1 weiß, dass Bieter 2 immer der Bietfunktion $\beta_2 = \frac{x}{4}$ folgt. Was ist eine beste Antwort von Bieter 1 in einer Erstpreisauktion? Was ist die beste Antwort von Bieter 1 in einer Zweitpreisauktion?

(e) Bieter 1 weiß, dass Bieter 2 immer der Bietfunktion $\beta_2 = x$ folgt. Was ist eine beste Antwort von Bieter 1 in einer Erstpreisauktion? Was ist die beste Antwort von Bieter 1 in einer Zweitpreisauktion?

5 Auktion theory, 22. 7. 2008

The exam was set at the University of Jena. Participants had 60 minutes to answer all questions. The exam sheet also contained the formulas from appendix A.

Consider an auction with two bidders. Valuations x are independently distributed and follow the density function

$$f(x) = \begin{cases} 2 - 2x & \text{if } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Assume risk-neutral preferences and symmetric equilibria. Please explain all your answers!

1. What is the equilibrium bidding strategy $\beta^{II}(x)$ in the second-price auction?

2. What is the expected payment $m^{II}(x)$ of a bidder with valuation x in the second price auction?

3. Use the revenue-equivalence principle to determine the equilibrium bidding strategy $\beta^{I}(x)$ in the first-price auction.

4. Use direct profit-maximisation (without using revenue-equivalence) to determine the equilibrium bidding strategy $\beta^{I}(x)$ in the first-price auction.

5. What is the equilibrium bidding strategy $\beta^{A}(x)$ in an all-pay auction?

6. Assume that the seller sets a fixed reverse price r. What is the expected payment $m^I(x, r)$ of a bidder with valuation x in a first-price auction?

7. What is the expected revenue of the seller in this case?

8. What is the optimal reserve price in a second-price auction?

A Formulas from the lecture:

$$\begin{aligned} F(x) &= \text{Prob}[X < x] \\ f(x) &= F'(x) \\ \int_{x_0}^{x^1} f(x)\,dx &= \text{Prob}[x_0 < x < x_1] \\ E(X|X < x) &= \frac{1}{F(x)} \int_0^x t f(t)\,dt \end{aligned}$$

$$\lambda(x) \equiv \frac{f(x)}{1-F(x)} \qquad \sigma(x) \equiv \frac{f(x)}{F(x)}$$

$$\begin{aligned} F(x) = 1 - \exp\left(-\int_0^x \lambda(t)\,dt\right) = \\ = \exp\left(-\int_x^\omega \sigma(t)\,dt\right) \end{aligned}$$

$$\begin{aligned} F_1^{(n)}(y) &= F(y)^n \\ f_1^{(n)}(y) &= n \cdot F(y)^{n-1} \cdot f(y) \\ f_2^{(n)}(y) &= f(y) \cdot F(y)^{n-2} \cdot (n-1) \cdot n \cdot \Big(1 - F(y)\Big) \\ f_2^{(n)}(y) &= f_1^{(n-1)} \cdot n \cdot \Big(1 - F(y)\Big) \end{aligned}$$

$$\begin{aligned} & f_{1,2}^{(n)}(y_1, y_2) = \\ & = \begin{cases} F(y_2)^{n-2} \cdot f(y_1) \cdot f(y_2) \cdot n \cdot (n-1) & \text{if } y_1 \geqslant y_2 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$f_2^{(n)}\left(z \middle| Y_1^{(n)} = y\right) = f_1^{(n-1)}\left(z \middle| Y_1^{(n-1)} \leqslant y\right)$$

$$\begin{aligned} m^{I}(x) &= G(x) \cdot E\left[Y_1^{(n-1)} | Y_1^{(n-1)} \leqslant x\right] \\ m^{II}(x) &= G(x) \cdot E\left[Y_2^{(n)} | Y_1^{(n)} = X\right] \\ \beta^{I}(x) &= E\left[Y_1 | Y_1 < x\right] = x - \int_0^x \frac{G(y)}{G(x)}\,dy \end{aligned}$$